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COMPETITIVE EQUILIBRIUM AND THE CORE IN CLUB ECONOMIES WITH ANONYMOUS CROWDING

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In club economies with anonymous crowding, competitive equilibrium states of the economy are efficient and coincide with the core when there is only one private good. Anonymous admission prices permit consumers with different tastes to share facilities. Consumers might do so when their demands for facility size and crowding coincide, in which case sharing is efficient. Thus, similarity of demands is the relevant consideration for efficiently grouping consumers with different tastes.

1. Introduction

Since Buchanan's (1965) insightful paper on clubs, many researchers[1] have discussed the idea that 'club theory' can be viewed as a branch of competitive equilibrium theory, even though club economies differ from private-goods economies through intragroup crowding and economies of scale in producing public goods. This paper investigates whether the major conclusions from the theory of competition in private-goods economies apply also to club economies with anonymous crowding.

In this introduction we briefly review what is known about the relationship between core and equilibrium in club economies with anonymous crowding. Section 2 presents a model. Section 3 shows that when consumers with different tastes have the same 'demands' for facility and crowding, core

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[1]For example Berglas (1976, 1981), Berglas and Pines (1981, 1984), Boadway (1982), Sandler and Tschirhart (1980, 1984), Scotchmer (1985a, 1985b), and Wooders (1978, 1980) discuss decentralization by prices in club economies with anonymous crowding. Berglas (1984), Wooders (1978, 1980), and Schweizer (1985) discuss cooperative equilibria (the core) with anonymous crowding.

states of the economy[2] (which are efficient) may permit people with different tastes to share the same facilities. This fact is used in Section 4, which discusses competitive equilibrium with anonymous prices.[3] With anonymous prices, consumers with different tastes will share facilities precisely when the core also permits this sharing.[4] Section 5 discusses the meaning of our results. An economy with a finite number of types is intended to approximate an economy in which all agents may differ. If all consumers have different indifference maps, consumers with different tastes *must* be grouped together for efficiency. Proposition 1 in section 3 shows that the *relevant* similarity of tastes in efficient groups is similarity of 'demands' for facility size and crowding. Beyond this, the shapes of their indifference maps are irrelevant.

The major ideas of interest from competitive theory are that (i) competitive equilibrium is efficient (the first welfare theorem), (ii) every efficient allocation is a competitive equilibrium for an appropriate distribution of endowments (second welfare theorem), (iii) the set of core allocations converges to the set of competitive allocations as the economy becomes large, and (iv) competitive equilibrium requires only anonymous prices, i.e. the price of a good is the same for all consumers.

Of the fundamental theorems of competitive economies, as applied to club theory with anonymous crowding, most attention has been given to the first, that competitive equilibrium is efficient. We argue this below for an equilibrium with anonymous prices by arguing that an equilibrium state of the economy is in the core and is therefore efficient. The argument is by revealed preference, that any assignment of goods and services preferred to the competitive equilibrium by all consumers is infeasible.[5]

[2]A state of the economy (or an 'allocation') is a partition of the consumers into clubs, and an allocation of public and private good within each club. When the intensity of club use, or number of visits per unit time, is variable, then the state of the economy also specifies each consumer's intensity of use. A state of the economy (and the implied utilities that consumers get) is in the *core* if no coalition of consumers, using only their own endowments, could form a coalition and improve their utility.

[3]When crowding is nonanonymous, which means that agents care about the types of other members as well as their numbers, efficient states of the economy cannot be supported as equilibria with anonymous admission prices. Scotchmer and Wooders (1987a) discuss equilibrium with nonanonymous admission prices (analogous to the equilibrium discussed here) in an economy with nonanonymous crowding.

[4]That prices can be anonymous has not been clear in the literature, because of the premise that consumers must be segregated by type in equilibrium. This premise suggests the construction of separate prices for each type of consumer. For example, Berglas and Pines (1983, pp. 148–149) define a price system, as a function of facility size and congestion, that is linked to the preferences of a homogeneous population. Such prices cannot be anonymous prices faced by all consumers. The anonymous price system defined below is linked to resource costs of providing the facility, and not to preferences.

[5]Most arguments in the literature that equilibrium is efficient are arguments from first-order conditions, rather than arguments from revealed preference. Such arguments require concavity. An exception is in Scotchmer (1985a), who presents a feasibility argument. A feasibility argument is also in Wooders (1978), although with a different equilibrium concept. The equilibrium concept for Buchanan clubs that is discussed in most of the literature has admission prices to

That competitive equilibrium is in the core occurs also in private-goods-exchange economies. However, in exchange economies with perfectly divisible goods and convex preferences, a core state of the economy may not be an equilibrium, unlike club economies, where *every* core state of the economy is an equilibrium.[6] The latter seems to occur because membership in a club is an indivisible commodity, and all surplus (relative to membership in another club) for consuming that commodity can be extracted through an admission price. A private-goods economy with a similar indivisibility is the assignment model in which there are integral numbers of houses and consumers. An equilibrium is an assignment of houses to consumers, and prices that equate demand and supply for each house. In the assignment model, every core state of the economy is an equilibrium, similarly to club models.[7]

Turning to existence of equilibrium, it follows from complete core equivalence that equilibrium exists if and only if the core is nonempty. We can see when the core is empty by characterizing the utility that consumers of a given type must achieve in the core. This is the utility that could be achieved in a homogeneous club with the facility size and intensity of use that maximize per-capita utility. We call this a 'type-optimal' club. In a core state of the economy, all consumers of a type must realize the utility available in a type-optimal club; otherwise, they could increase their utility by forming a type-optimal club with others of their type. Clubs shared by consumers with different tastes may sometimes achieve core utilities, but mixing types cannot strictly increase utility of any type above that realized in a type-optimal club.[8] The core is empty if the population cannot be partitioned into clubs that give each consumer the utility he would realize in his type-optimal club. (This does not mean that consumers *must* be partitioned into homogeneous type-optimal clubs in core states of the economy.)

clubs and no per-unit (or Lindahl) prices, and firms in equilibrium make zero profit. This deters entry. In Wooders' equilibrium, consumers pay no admission fee, but pay an anonymous price per unit for public goods once they are in the club. With such prices, clubs may make positive profit, which is then distributed to members. Ellickson (1979) interprets efficiency by showing a formal similarity between clubs and indivisible commodities in a standard model of exchange. The associated argument for efficiency is also a feasibility argument.

[6]This is provided the population size of each type exceeds the minimum size of membership that maximizes per-capita utility and there is only one private good that can be transferred between clubs. If a homogeneous population were of size less than required to maximize per-capita utility, then the core may require everyone of that type to be in the same club, and the private good remaining after paying for the shared facility could be distributed so that different agents of the same type have different utilities. Such an allocation could not be an equilibrium with anonymous prices, since each consumer with the same tastes and endowment has the same opportunities and must therefore get the same utility in equilibrium. If the population size of a type exceeds minimum efficient scale for that type, then the core, if it exists, must assign the same utility to all consumers of that type. See Wooders (1978, theorem 3).

[7]Cf. Kaneko (1983).

[8]The point that mixed clubs cannot provide strictly more utility than segregated clubs has been made by Berglas and Pines in several papers, and we include it in Proposition 1 for completeness. Our argument is a feasibility argument.

Emptiness of the core is easiest to see with a homogeneous population. Suppose endowments are w, and $n(w)$ is the group size that maximizes per-capita utility. Then, provided the population size N is greater than $n(w)$, the core is empty unless $N=kn(w)$ for integer k.[9] Otherwise, some consumers do not achieve their core utilities. The size $n(w)$ is sometimes called the 'minimum efficient scale' and is usually assumed positive and finite. This is motivated by the idea that eventually economies of scale from sharing a public good are outweighed by the crowding costs – gains to scale are exhausted by groups of size $n(w)$.

Since competitive equilibrium states of the economy are in the core, competitive equilibrium does not exist when the core is empty. This is true also in exchange economies. However, unlike club economies, the convexity of preferences suffices to ensure nonemptiness of the core in exchange economies. Since the group consisting of all agents is optimal in an exchange economy, the problem of partitioning the population into type-optimal groups does not arise. In contrast, core states of a club economy typically require more than one group. In club economies, the core may well be empty even when preferences are concave and costs are convex.[10]

Although the core may be empty for every distribution of endowment, an efficient state of the economy will generally exist. If an efficient state of the economy is in the core relative to some distribution of endowment, then it can be supported as an equilibrium and the second welfare theorem holds.

In large club economies, emptiness of the core, nonexistence of equilibrium and failure of the second welfare theorem may not be severe problems, provided we are willing to consider 'approximate' core and equilibrium states of the economy. We postpone discussion of approximations to section 5, since it refers to our discussion in sections 3 and 4 of when the core permits consumers with different tastes to share clubs.

2. The model

We shall discuss clubs with variable intensity of use, as in Berglas and

[9]Game theorists use the term 'balanced game' to refer to a game (a set of players and payoffs to coalitions) with the property that the core is nonempty. In the club-theory literature, emptiness of the core is called the 'integer problem'. The club-formation game is balanced if there is no integer problem.

[10]Conditions can be identified under which the integer problem disappears. For example, if maximum per-capita utility is achieved for type i by group-size n^{*i} and also $n^{*i}+1$, and this is true for all types, then for large enough finite populations the core is nonempty. See Wooders (1978, theorem 3.) It follows that competitive equilibrium exists. The analogous assumption with divisible agents is that maximum per capita utility is achieved on an interval rather than at a point. A different approach to existence recognizes that in a finite economy with minimum efficient scale, firms will not be price-takers. Equilibrium in an oligopoly may exist even when competitive equilibrium does not. See Scotchmer (1985a, 1985b) who discusses equilibria in which firms compete in prices, and quality (congestion) is endogenous to prices.

Pines (1981, 1984), Sandler and Tschirhart (1984), and Scotchmer (1985b). Buchanan (1965) clubs are a special case of this model, with intensity of use restricted to be one. For simplicity, we shall assume there are two types of consumer, a and b. A club has facility of size X, and type i consumers make visits in amount v^i per unit time. Preferences are defined on the consumption space $[m, X, v, c]$,[11] private good, facility size, number of visits, and congestion, which is the total number of visits per unit time. We assume that utilities are strictly increasing in the private good. If different types of consumers visit the facility, congestion is $c = \Sigma n^i v^i$, where n^i is the number of type i making visits. Crowding is anonymous in that only the number of visits, and not who visits, affects utilities and cost. Since there may be a cost for building the facility and additional costs per visit, the cost function is written $C(X, c)$. There is an economy-wide endowment of private good W which can be consumed as private good or used to pay for club goods. The economy-wide endowment W can be divided between the types according to $N^a w^a + N^b w^b = W$, where w^i is the endowment to an agent of type i, $i = \text{a, b}$, and N^i, $i = \text{a, b}$, are the numbers (measures) of agents of the different types.[12]

A *state of the economy* is a partition of the population into clubs, with a facility provided to each club, an allotment of private good to each consumer, and a number of visits for each consumer in each club. We discuss 'equal treatment' states of the economy in which all consumers of the same type receive the same utility.[13]

3. The core and efficient allocations

A *core state of the economy* is one with the property that no group of consumers could provide each member with more utility, using only their own endowments.[14]

[11] A special case is that preferences are defined on $[m, v, h(c/X)]$, where $h(\cdot)$ is some function, as in Sandler and Tschirhart (1984).

[12] Most of the referenced literature assumes populations are continuously divisible. Otherwise differentiable methods for describing efficient allocations are inappropriate. Letting people be divisible emphasizes that the 'integer problem' is not about the fact that individuals are integral, but rather that the number of clubs is integral and gains to scale are exhausted with a finite number of members. Everything we say in this paper applies both to divisible people and integral people unless we specify otherwise.

[13] There may be Pareto optima without the equal-treatment property, but all competitive equilibria will have equal treatment, since consumers of the same type have the same endowment as well as the same preferences. States of the economy in the core will also have equal treatment, provided the population of type i exceeds minimum efficient scale, as mentioned above.

[14] When there is more than one private good, a subtlety arises in defining the core. By definition of what it means for a coalition to 'block', trade between a deviating coalition and the rest of the economy is not allowed. Then, in order to allow trade of private goods among members of different clubs, we must allow a coalition to contain many clubs, whose members can trade private goods freely. With one private good, we can assume a coalition is a club, since no trade occurs between clubs.

Our discussion here follows most of the club literature in that we define the facility size and crowding which would be preferred by each type, if there were enough people with those preferences and endowments to have a homogeneous club. We will then use this definition to characterize the core and equilibrium. The maximum utility achievable in a segregated club is (where, for the segregated club with equal treatment, congestion c is simply nv)

$$u^i(w^i) = \max_{(X,v,n)} U^i\left[w^i - \frac{C(X,c)}{n}, X, v, c\right]. \tag{1}$$

The solution is labeled $[X^i(w^i), v^i(w^i), n^i(w^i)]$ or $[X^i(w^i), c^i(w^i)]$, where $c^i(w^i) = n^i(w^i)v^i(w^i)$.[15]

We shall use the term 'type-optimal' to refer to the facility size and congestion $[X^i(w^i), c^i(w^i)]$ in segregated clubs which achieve the maximum per-capita utility $u^i(w^i)$. Fig. 1 illustrates the core-utility frontier that would be achieved if all consumers could be accommodated in type-optimal clubs (that is, if there were no integer problem). For each division of endowment, (w^a, w^b), we have graphed $[u^a(w^a), u^b(w^b)]$.

This utility frontier can be achieved if, for each type of consumer i, $N^i = kn^i(w^i)$, for an integer k. When the integer condition is not satisfied, only points inside the frontier are feasible, and efficiency will sometimes *require* (not simply permit) mixed clubs.[16] With anonymous crowding, points to the 'northeast' of the utility frontier cannot be achieved in mixed or segregated clubs.

States of the economy in the core require that each consumer achieves the utility available in a type-optimal club, $u^i(w^i)$, provided $N^i > n^i(w^i)$. Otherwise

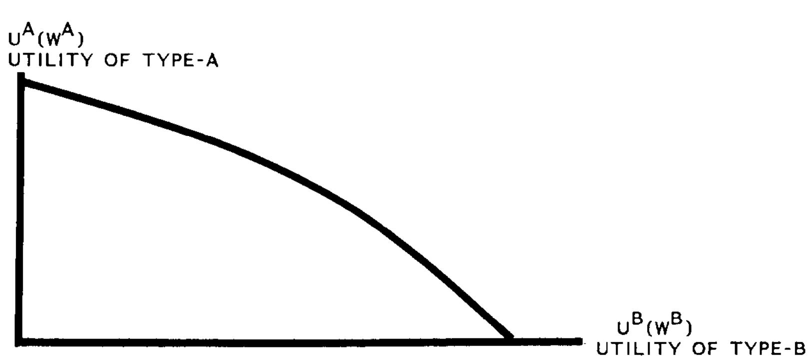

Fig. 1. Core-utility frontier.

[15]This solution, of course, may not be unique. Our arguments below are valid whether or not the solution is unique, but for ease of exposition we assume uniqueness.

[16]This is discussed by Sandler and Tschirhart (1984).

the core is empty, because any consumer not achieving $u^i(w^i)$ could bribe other consumers to join him in creating a type-optimal club. Thus, fig. 1 illustrates utilities that must be achieved in the core.

Fig. 2 graphs an example of type-optimal clubs for varying endowments. For each endowment, type-i has a preferred facility size $X^i(w^i)$, and a preferred amount of congestion, $c^i(w^i)$. These points are plotted for each endowment w^i.[17] Proposition 1 below states that if the curves depicted in fig. 2 intersect at a feasible division of total endowment, then the same utilities can be achieved in mixed clubs as in segregated clubs. For such endowments, types a and b have the same 'demand' for facility and for congestion, and the core may have mixed clubs. Population sizes, N^a and N^b, may even be such that mixed clubs are required to achieve core utilities, as in Example 1 below.

Proposition 1.[18] *Suppose consumers are divisible.*[19] *Core utilities can be achieved in mixed clubs if and only if* $[X^a(w^a), c^a(w^a)] = [X^b(w^b), c^b(w^b)]$. *If core*

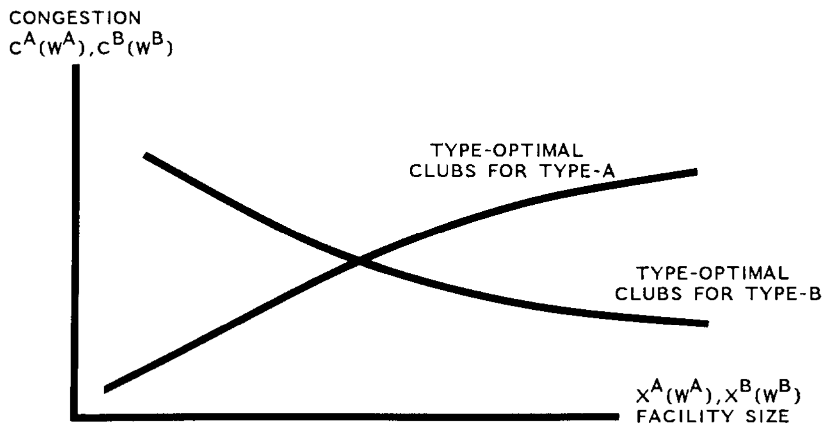

Fig. 2. Type-optimal clubs.

[17]The curves need not be as well behaved as in fig. 2. Different endowments may require different facility sizes but the same membership, or different endowments may require the same facility size and membership.

[18]This proposition addresses a discrepancy between how Berglas and Pines (1981) state their proposition 2 and what they prove. They state that segregated clubs must be strictly superior to mixed clubs when the integer condition is satisfied. But their argument does not support this, and in fact their argument does not conflict with the result presented here.

[19]When consumers are integral, this theorem needs an additional technical condition, which is simple but messy. To see this, suppose the type-optimal facility size and congestion are [10, 30] for types a and b, and that $v(w^a)=6$, $v(w^b)=5$. Then the numbers of types a and b that are required for type-optimal clubs are 5 and 6, respectively. For no fractions $(f, 1-f)$ is it true that $5f$ and $6(1-f)$ are both integral. Hence we cannot form a mixed club with integral people. Since the required additional conditions are straightforward and add little insight, we omit them.

utilities can be achieved in mixed clubs, they can also be achieved in segregated clubs.

Proof. [If]. Suppose the initial endowments are (w^a, w^b), and $[X^i(w^i), c^i(w^i)] = [X^*, c^*]$, $i = a, b$. Suppose we construct a mixed club as follows. Give to $i = a, b$, private good in amount $m^i = w^i - C(X^*, c^*)/n^i(w^i)$. Construct a club with $fn^a(w^a)$ people of type a, where $0 \leqq f \leqq 1$, and with $(1-f)n^b(w^b)$ people of type b. (When $f=0$ or $f=1$, we have segregated clubs.) Allow each type a person to make $v^a(w^a)$ visits and each type b person to make $v^b(w^b)$ visits. Produce facility in amount X^*. Then since congestion in the constructed club is $c^* = fn^a(w^a)v^a(w^a) + (1-f)n^b(w^b)v^b(w^b)$, for any f this club gives to each person exactly the same bundle $[m^i, X^*, v^i(w^i), c^*]$ as he has in a segregated club and thus provides the same utility.

The constructed club is feasible because it uses the same amount of resource per person of each type as segregated clubs use. The total resource available is $fn^a(w^a)w^a + (1-f)n^b(w^b)w^b$. The total resource used is $fn^a(w^a)m^a + (1-f)n^b(w^b)m^b + fn^a(w^a)C(X^*, c^*)/n^a(w^a) + (1-f)n^b(w^b)C(X^*, c^*)/n^b(w^b)$, or $fn^a(w^a)m^a + (1-f)n^b(w^b)m^b + C(X^*, c^*)$, which equals the resource available. Consequently, for any composition of the clubs (for any f), the mixed clubs we have constructed are feasible.

But mixing cannot achieve strictly greater utility than segregated clubs. Suppose a mixed club achieves core utilities and is described by $[X^*, m^{*a}, m^{*b}, v^{*a}, v^{*b}, Q, t]$, $0<t<1$, where X^* is the facility size, m^{*i} are the amounts of private goods consumed in the mixed clubs by $i=a,b$, v^{*i} are the visits made by $i=a,b$, Q is the total number of members of the club, tQ is the number of type a and $(1-t)Q$ is the number of type b. Then $c^* = tQv^{*a} + (1-t)Qv^{*b}$. We show that the utilities achieved in the mixed club are achievable in segregated clubs. Choose endowments $w^{*i} = m^{*i} + C(X^*, c^*)v^{*i}/c^*$. These are feasible, since $tQw^{*a} + (1-t)Qw^{*b}$ equals $tQm^{*a} + (1-t)Qm^{*b} + C(X^*, c^*)$, which is the total resource used by the mixed club. But then segregated clubs with membership $n^{*i} = c^*/v^{*i}$, $i=a,b$, visits v^{*i} and facility size X^* are feasible and achieve the same utility as the mixed clubs. These segregated clubs are feasible because $n^{*i}w^{*i} = n^{*i}m^{*i} + C(X^*, c^*)n^{*i}v^{*i}/c^* = n^{*i}m^{*i} + C(X^*c^*)$. In the segregated clubs we have constructed, each person consumes the same bundle $[m^{*i}, X^*, v^{*i}, c^*]$, as he consumed in the mixed club and therefore gets the same utility.

[Only if]. The previous argument showed that $u^i(w^{*i})$ is at least as large as the utility achieved by type i in the mixed club. Suppose, contrary to the proposition, that $[X^*, c^*]$ (the facility size and congestion in the mixed club) are not optimizers that achieve $u^i(w^{*i})$. Then $u^i(w^{*i})$ exceeds the utility in the mixed club for this i and we have thus made a strict improvement by segregating consumers. This contradicts the premise that the mixed club achieved core utilities. We conclude that $[X^*, c^*]$ must be optimizers for $u^i(w^{*i})$, $i=a,b$. Q.E.D.

The condition under which mixing may achieve core utilities in Buchanan clubs is that $[X^i(w^i), n^i(w^i)]$ are the same for $i = \mathrm{a,b}$. The argument is identical to Proposition 1, except that v^i must be restricted to equal one.

Since (as we show below) states of the economy in the core are efficient, and since we have stated the proposition with endowments as parameters, Proposition 1 describes distributions of income for which efficiency may permit mixed clubs.

We close this section with two examples, showing that the core may permit mixing consumers with different tastes, even when tastes are sufficiently different that indifference surfaces cross everywhere in the consumption space. The first example is with Buchanan clubs and the second example has variable intensity of use, or number of visits. In both examples, indifference surfaces cross everywhere, provided the parameters for types a and b differ.

Example 1. Preferences are defined by $U^i(m, X, n) = mX^{\alpha_i} n^{-\beta_i}$ and $C(X, n) = k + qX^2$, where k and q are constants. Restrict $\alpha_i/(2\beta_i - \alpha_i) = \rho > 0$ for $i = \mathrm{a,b}$. Then $X^{*2} = \rho k/q$ and $n^i(w^i) = (k/w^i)\ [1 + \rho(2 + \alpha_i)/\alpha_i]$. Clearly, we can choose $(w^{\mathrm{a}}, w^{\mathrm{b}})$ such that $n^{\mathrm{a}}(w^{\mathrm{a}}) = n^{\mathrm{b}}(w^{\mathrm{b}})$. Suppose that $n^{\mathrm{a}}(w^{\mathrm{a}}) = n^{\mathrm{b}}(w^{\mathrm{b}}) = 10$, and that there are 15 of both a and b. Then the core is nonempty and requires at least one mixed club.

Example 2. Suppose the only feasible facility size is $X = 1$.[20] Suppose, when $X = 1$, that $U^i(m, v, c) = m + s^i \log(v) - t^i c$ and $C(X, c) = 1$. Restrict the constants so that $s^i/t^i = r$, $i = \mathrm{a,b}$. Then for all w^i, $[X^i(w^i), c^i(w^i)] = [1, r]$, and Proposition 1 applies.

4. Competitive equilibrium

In order to discuss competitive equilibrium, we must have a complete price system. Otherwise it is impossible to evaluate whether each firm is at a profit maximum on its production set. The feature of this price system that we wish to emphasize is that firms do not need to discriminate among consumers in order to achieve efficiency. Prices are anonymous, and if it is optimal to segregate consumers (which, by Proposition 1, it may not be), then this will happen by self-selection on the part of consumers.

The good is a visit to a club of particular quality, (X, c). Thus, a complete price system is a price function $P(X, c)$ assigning to each good (visit of

[20]This example focuses on the difference in preference for visits, rather than the difference in preference for facility. It should be clear from the two examples together that a more complicated example could be constructed, focusing on both facility size and intensity of use simultaneously.

quality (X,c)) a price.[21] A competitive equilibrium is an endowment for each consumer (types a and b), a state of the economy (each consumer visits some club, consumes some amount of private good, and makes some number of visits), and a price system. The usual utility representation for this economy imposes the restriction that each consumer visits only one club. (Otherwise, the utility function would contain many three-tuples (X,v,c).)

The following definition of competitive equilibrium is similar to the definition of competitive equilibrium for a private-goods economy in that there is a complete price system, each firm takes prices $P(X,c)$ as given,[22] and each firm chooses a profit-maximizing (X,c) from its feasible production set, paying cost $C(X,c)$.[23] This competitive equilibrium differs from competitive equilibrium in a private-goods economy in that, when the firm chooses the supply of visits, c, it is also setting one aspect of quality, congestion. The other aspect of quality is the facility size, X. In competitive equilibrium, demand equals supply in the sense that each firm supplies its preferred number of visits, and no consumers are queued outside waiting to get in.

Competitive equilibrium may fail to exist because of the integer problem. At competitive (average-cost-per-visit) prices, consumers will want to be in their type-optimal clubs (or possibly in mixed clubs with the same facility size and congestion level as type-optimal clubs), but the population size may not be an integer multiple of the type-optimal club membership, $n^i(w^i)$. In such a case, demand will not equal supply at competitive prices.

Definition. A competitive equilibrium is a triple $[\{w^i\}, P(X,c), \{v^i(X,c)\}]$ such that:

$$\text{if } v^i(X,c)>0, \text{ then}$$

$$U^i[w^i - v^i(X,c)P(X,c), X, v^i(X,c), c] \geqq U^i[w^i - \bar{v}P(\bar{X},\bar{c}), \bar{X}, \bar{v}, \bar{c}],$$

$$\text{for all } (\bar{X},\bar{v},\bar{c}); \tag{2}$$

$$\text{zero-profit anonymous prices: } cP(X,c) - C(X,c) = 0. \tag{3}$$

Condition (2) says (i) at the visited facility, the number of visits maximizes utility at the facility's price[24] and (ii) no other potential or actual zero-profit

[21] The complete set of prices for Buchanan clubs would be $P(X,n)$, a price for membership in a type (X,n) club.

[22] Berglas and Pines (1981) point out that the analogy with competitive theory requires that firms be price-takers and face a complete price system. We add to their observations that the price system can be anonymous.

[23] The protection set for each firm is $\{(X,c,-C) \mid (X,c) \geqq 0, \; -C \leqq -C(X,c)\}$. The minimum cost of producing c visits of quality (X,c) is $C(X,c)$.

[24] Otherwise, we would not have $U^i[w^i - v^i(X,c)P(X,c), X, v^i(X,c), c] \geqq U^i[w^i - vP(X,c), X, v, c]$ for all v, where (X,c) is the visited facility.

facility (X,c) would yield higher utility than the chosen facility. Condition (3) says that competitive prices will be the average cost per visit, since that price yields zero profit. Equilibrium profit must be zero to deter entry.

There is no presumption in the definition that the visits in each firm are made by one type of consumer or another. The definition merely states that if consumer i visits a type (X,c) facility, he pays price $P(X,c)$ per visit to do so. Since we have defined only visit prices, the following proposition implies that competitive firms do not need to charge lump-sum entry fees in order to achieve efficiency.[25]

The following proposition states that there is complete equivalence between core and competitive states of the economy. This contrasts with private-goods economies in which typically the core strictly contains the set of equilibrium states.

Proposition 2. Every competitive equilibrium state of the economy achieves core utilities and every core state of the economy is a competitive equilibrium, provided that (for all consumer types i) the number of type i exceeds the minimum number of type i required for a type-optimal club.

Proof. The reason we need $N^i \geqq n^i(w^i)$ is that otherwise the core may not give the same utility to consumers of the same type. Since competitive equilibrium must always give the same utility to consumers with the same preferences and endowments, a state of the economy in the core may not be a competitive equilibrium.

We showed in Proposition 1 that core utilities can always be achieved in segregated clubs. Hence, we can always describe the core utility for type i as the maximum of (1), even though the core utility may be achieved in a mixed club. We can equally well describe this core utility as achieved by the (X,v,c) that maximizes $U^i[w^i - vC(X,c)/c, X, v, c]$. But since $C(X,c)/c = P(X,c)$, the latter expression is the same as $U^i[w^i - vP(X,c), X, v, c]$, which the consumer maximizes by choice of v and of (X,c). Consumers choose their consumption so as to solve the maximization problem that defines core utilities. Thus, type i achieves the same utilities in competitive equilibrium and in the core. Q.E.D.

The first welfare theorem, that competitive equilibrium is efficient, now follows from the fact that core allocations are efficient.[26]

[25]This issue is discussed in Berglas and Pines (1981, 1984) and in Sandler and Tschirhart (1984). The absence of lump-sum fees is to be contrasted with the case of imperfectly competitive firms in the same club model. Compare with Scotchmer (1985b), who discusses price competition among a few imperfectly competitive firms and shows that lump-sum fees will be charged. Furthermore, the lump-sum fee is a measure of market power. In the competitive case, firms do not have market power, and thus there is no lump-sum fee.

[26]In superadditive games, efficiency of the core follows from the fact that if it were feasible to increase some agent's utility without decreasing others' utilities, the coalition of the whole would agree to make this improvement. If a coalition can subdivide into several clubs, then we have

Proposition 3 (First Welfare Theorem). *States of the economy in the core are efficient, and therefore competitive equilibria are efficient.*

Proof. Suppose a state of the economy in the core was not efficient. Then it would be possible to give more utility to type a or type b than is achieved in the core. Proposition 1 tells us that strictly greater utility cannot be achieved by mixing types a and b in clubs.[27] Therefore, if we are to give more utility to type a or b, we must do so by giving that type more endowment. But with fixed aggregate resources, an increase in the endowment of one type requires a decrease in the endowment given to the other type. Since $u^i(w^i)$ is strictly increasing, we can only increase type a's utility by decreasing type b's utility. Q.E.D.

None of these arguments requires that consumers actually be segregated in type-optimal clubs, but only that mixed clubs cannot strictly improve on utility of both types. If core utilities are achievable in mixed clubs, as when initial endowments satisfy the condition expressed in Proposition 1, then clubs in a core allocation may well be mixed. Likewise, clubs in competitive equilibrium may be mixed. Consumers who share the same facility by definition get the same facility size and congestion, (X,c), but the members who in aggregate contribute to congestion level c may have different tastes and may have different intensities of use, v^i.

Efficient states of the economy can, of course, be defined even when consumers cannot be accommodated in type-optimal clubs. Some consumers will achieve less utility than would be available in type-optimal clubs. The economy is inside the core-utility frontier in fig. 1. Suppose that in an efficient state, a consumer of type i makes v^* visits to a facility of type (X^*,c^*) and consumes private good in amount m^*. If such a consumer is not in a type-optimal club, he achieves less utility than $u^i[m^*+v^*C(X^*,c^*)/c^*]$. The latter is the core utility the consumer would achieve in a type-optimal club if he were simply given his resource cost, $m^*+v^*C(X^*,c^*)/c^*$, as initial endowment. Since consumers with this initial endowment will want to consume club facilities other than (v^*,X^*,c^*) – they will want to be in type-optimal clubs – this efficient state will not be an equilibrium.

Proposition 4 (Second Welfare Theorem). *Consider an efficient state of the economy in which consumer i has consumption* $[m^{*i},v^{*i},X^{*i},c^{*i}]$ *and achieves*

superadditivity. But if a coalition is a club, then economies may not be superadditive, since the per-capita utility achieved by a group of size $2n^i(w^i)$ may be less than (not greater than) the utility achieved by two groups of size $n^i(w^i)$. Hence, the coalition of the whole in one club will not be efficient, and the argument that the core is efficient is slightly more subtle.

[27]Since the core is assumed nonempty, the integer condition is satisfied and we do not need the qualification that mixing may be strictly superior when it is infeasible to partition the population into type-optimal clubs.

utility $u^i[m^{*i}+v^{*i}C(X^{*i},c^{*i})/c^{*i}]$. *Then this efficient state of the economy achieves utilities in the core for endowments* $w^i=m^{*i}+v^{*i}C(X^{*i},c^{*i})/c^{*i}$, *and therefore is a competitive equilibrium state of the economy.*

Proof. With the endowments constructed, every consumer achieves the utility he would achieve in a type-optimal segregated club. Such utilities are in the core relative to the endowments $m^{*i}+v^{*i}C(X^{*i},c^{*i})/c^{*i}$ for each i, and by Proposition 2, the core state of the economy is a competitive equilibrium. Q.E.D.

5. Conclusion

We conclude with some remarks on the existence problem and the significance of our 'mixing theorem', Proposition 1.

The Introduction mentioned that, because of the integer problem, the core of an economy with a finite number of types may be empty. But as the population becomes large, feasible utilities can become close to core utilities.[28] Thus, even though the core will typically be empty, an 'approximate core' or 'ε-core' will exist. A state of the economy in the ε-core has the property that no deviating coalition could increase each member's utility by at least ε.[29] There will also exist an approximate competitive equilibrium with the properties that no firm could increase profit by more than ε by changing output, and no consumer could increase utility by more than ε with his given endowment.

The 'types' economy is intended to approximate the real case of interest, that no two consumers may be exactly alike. Our 'mixing theorem', Proposition 1, suggests how to group consumers in the approximate core when all agents differ: our conjecture is that consumers' *demands* for facility size and crowding must be similar in each club, even though their endowments and preferences may be very different. If all consumers differ, each club will be heterogeneous, but nevertheless, in a large economy, each consumer's utility can be very close to the utility achievable in a type-optimal (homogeneous) club. The empirical content of Proposition 1 is that it cannot be assumed that heterogeneous communities imply nonoptimality.

The notion of letting the population get large is not straightforward when

[28]An argument given by Wooders (1980) for Buchanan clubs is essentially this: for each type, form as many groups of the type-optimal size as possible. Then transfer private good from agents in the type-optimal clubs to the 'leftover' agents until utilities of all agents of the same type are equalized. Although this utility will be less than the core utility, the reduction will be small when the population is large, since the number of 'leftovers' is small relative to the number of each type. Schweizer (1985) considers clubs that may operate part time at efficient size, i.e. fractional clubs. These two approaches have different interpretations, but are technically virtually equivalent; this is especially easy to see in the transferable utility case considered by Schweizer.

[29]There are several ways to define approximate cores, cf. Wooders (1980, 1987), Wooders and Zame (1984), and Scotchmer and Wooders (1987b).

all agents differ, since we cannot simply replicate the set of agents. (We would then have types.) One approach, used by Wooders and Zame (1984) for large games and transferable utility, is to posit a space of endowments and attributes that determine possible outcomes. Each consumer is a point in this space. Enlarging the economy means increasing the number of agents. This could be done with a fixed probability measure on the space of endowments and attributes. With probability one,[30] populations will not duplicate any agents. However, in larger economies it will be easier to assemble groups of agents with similar demands for crowding and facility. (The shapes of their indifference maps in other respects are irrelevant.) Therefore, we conjecture that in such a model, utilities achievable in an approximate core will converge to core utilities as the economy becomes large.[31] This is despite the fact that clubs need not be 'homogeneous' (in the sense that the indifference maps of all members are close) even in the limit.

[30]Provided the measure is atomless.

[31]Suppose, for example, that a taste parameter θ is distributed uniformly. Suppose that a type (w, θ) consumer has demands $[n(w,\theta), X(w,\theta)] = [n^*, \theta]$, independent of endowment w. (This is a Buchanan club; the intensity of use does not vary.) For each population size, partition the domain of θ into intervals, each containing n^* people. The consumers in an interval will share a facility of size somewhere in the interval. As the number of consumers chosen from this distribution grows, the size of the interval required to assemble a group of size n^* becomes smaller. Since the consumers in a group become more similar in taste, their taste for facility can be more closely satisfied.

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