

# Property Value Maximization and Public Sector Efficiency

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This paper analyses the efficiency implications of property value maximization. Communities are open, so that utilities are parametric to housing producers and the local government. Each local government chooses its public good output to maximize aggregate property value in the community, ignoring feedback effects on the composition of the housing stock. It is shown that this type of government behavior generates an equilibrium in which all communities are internally Pareto-efficient.

## 1. INTRODUCTION

It is now generally recognized that in the absence of actively optimizing governments, the equilibrium states of an economy with local public goods need not be Pareto-efficient. Recent analysis shows that when governments are passive (as under majority rule), inefficient equilibria may persist even in the presence of free mobility and perfect information (see, for example, Bewley [1], Stiglitz [11], and Brueckner [3]). These conclusions probably would not have surprised Tiebout, who argued several decades ago [12] that a type of entrepreneurial behavior on the part of local governments is required to achieve efficiency. In particular, Tiebout believed that if each government attempted to maintain its community's population at the level consistent with the minimum per capita cost of (optimal) public consumption, then an efficient equilibrium would emerge as consumers voted with their feet. Although Tiebout's argument was suggestive, dissatisfaction with the vagueness of his entrepreneurial assumption and general style of analysis has led researchers to formulate more precise models. While the recent negative verdict on efficiency with passive governments was one byproduct of greater precision, another line of research has been directed toward refining the notion of entrepreneurial government behavior. In two significant contributions, Sonstelie and Portney [9] and Wooders [14] attempted to show how profit-maximizing behavior on the part of local governments might lead to efficient equilibria. In Sonstelie and Portney's analysis, governments are portrayed as producing housing as well as providing a public good (the government is in effect the community developer), so that profit equals aggregate house rent less housing and public sector production costs. In Wooders' model, governments are not in the housing business, so that profit is simply equal to tax revenue less public sector costs.

The present paper adds to this tradition by analyzing the efficiency implications of a related type of entrepreneurial government behavior: property value maximization. In the analysis, governments are imbedded in an economy where private housing and land markets operate, and government budgets are required to balance. The resulting model is therefore institutionally realistic, unlike those discussed above. The government's objective function, aggregate property value, is the difference between aggregate house rent and public sector costs. In choosing its public good output to maximize aggregate value, the government takes consumer utilities as parametric (each community is open to migration), and ignores the ultimate influence of its public good choice on house sizes and land rent, focusing solely on the change in aggregate value caused by capitalization of rent and tax changes for existing houses. The main result of the paper is that this type of property value maximization generates an equilibrium where each community is internally Pareto-efficient. While this result holds strictly when revenue is raised by a "house tax" (a head tax on each house owner), the weaker statement that communities are internally efficient conditional on nonoptimal housing choices applies when a (distortionary) property tax is in force. It is shown by example that even though internal efficiency is guaranteed by property value maximization, the community system equilibrium as a whole may be globally inefficient (the assignment of consumers to communities may be wrong).

As should be clear from the above discussion, community profit in Sonstelie and Portney's analysis is simply aggregate property value (rents minus public sector costs) less housing production costs. Indeed, Sonstelie and Portney recognize that property value and profit maximization are equivalent when house sizes (and thus housing costs) are fixed. The similarity of governmental objective functions, however, should not mask the considerable differences between the present analysis and that of Sonstelie and Portney. While a crucial element of their model is an hedonic-type price function relating rent to house size and public consumption, the analysis is seriously incomplete because the difficult and subtle issue of how this function is generated is entirely ignored. In contrast, the present paper explicitly derives house rents from consumer bid-rent functions under the assumption of parametric utilities. This approach was used in an earlier paper (Brueckner [4]) to show the connection between property values and efficiency in the context of an empirical model. Here, the bid-rent approach is applied to yield a government decision rule.<sup>1</sup>

<sup>1</sup>It should be noted that Edelson [5] and Wildasin [13] consider models where individual residents of a community vote for the public good level which maximizes the value of their own property. While Edelson was concerned with showing the circumstances under which voters agree on the optimal public good level, he mentions in passing (and without proof) that aggregate property value maximization is Pareto-optimal. Wildasin's discussion, which also

The next two sections of the paper present the analysis of property value maximization under the house tax and property tax regimes. The last two sections discuss global efficiency and present conclusions.

## 2. PROPERTY VALUE MAXIMIZATION WITH A HOUSE TAX

The analysis will focus on a single open community which is part of a large community system. It will be assumed that all consumers have identical endowments of the numeraire commodity  $x$ , and that the rent from the community's fixed land area is divided equally among its residents. Although it could be assumed that each consumer owns land in a number of communities in the system, the assumption of land rent sharing among the (current) residents of each community allows a straightforward analysis of Pareto-efficiency, as will be seen below.

In addition to his consumption of the numeraire, each resident consumes the public good  $z$  and housing  $q$ . The treatment of housing in the analysis follows Rosen [8], with each consumer viewed as buying an indivisible bundle of housing rather than acquiring the commodity at a fixed price per unit, as would occur with a homogeneous good. For simplicity, it is assumed that individual housing bundles are produced with constant returns to scale using inputs of land and  $x$  (decreasing returns could be imposed without changing any of the paper's results). The public good is produced with a non-increasing-returns technology whose sole input is  $x$ . Public good congestion is allowed, so that the  $x$  input required to generate a given public consumption level may increase with the population  $n$  of the community, but the analysis in no way depends on this assumption.

Consider first the input choices of the housing producer. Letting  $r$  denote land rent per acre, and  $l$  and  $x^h$  denote housing inputs of land and  $x$  respectively, the Lagrangean expression for the producer's cost minimization problem is

$$x^h + rl - \lambda(H(x^h, l) - q), \quad (1)$$

where  $H$  is the production function and  $q$  is the size of a given house. The first-order conditions yield

$$\frac{H_2(x^h, l)}{H_1(x^h, l)} = \frac{H_2(x^h/l, 1)}{H_1(x^h/l, 1)} = r, \quad (2)$$

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yields an efficiency result, suffers from the same defect as Sonstelie and Portney's: a housing price function is used without discussion of where it comes from. In another study, Negishi [7] shows the efficiency of an extremely restrictive type of land value maximization in a model based on very special assumptions.

where the first equality follows from the first degree homogeneity of the production function. Letting  $S$  denote  $x^h/l$ , (2) implies that  $S$  is a function of  $r$ ,  $S(r)$ . Recalling that the Lagrange multiplier  $\lambda$  is marginal (and average) production cost, and noting the first-order condition  $\lambda = 1/H_1(x^h, l)$ , marginal cost may be written as

$$\frac{1}{H_1(S(r), 1)} \equiv a(r). \quad (3)$$

For future reference, note that since  $q = lH(x^h/l, 1) \equiv lh(x^h/l)$ , the land input for a house of size  $q$  may be written as

$$l = q/H(x^h/l, 1) = q/h(S(r)).$$

The next step in the analysis is to derive the rent for a house of size  $q$  which leaves the producer a given amount of profit after payment of the "house tax" levied to finance the public good. Note that the producer, not the consumer, pays the tax. Let  $C(z, n)$  denote the cost function for the congested public good, which satisfies  $C_1 > 0$ ,  $C_{11} \geq 0$ , and  $C_2 \geq 0$  ( $C_2 \equiv 0$  corresponds to a pure public good). It follows that a house tax of  $C(z, n)/n$  levied on the owner of each dwelling in the community will balance the local government's budget. Letting  $F$  denote house rent, it then follows that the producer will earn a profit of  $\pi$  when  $F$  satisfies  $F - a(r)q - C(z, n)/n = \pi$ . The required rental payment as a function of  $q$ ,  $z$ ,  $r$ , and  $\pi$  may therefore be written

$$F = \pi + a(r)q + C(z, n)/n. \quad (4)$$

Equation (4) defines a family of iso-profit rent contours, constructs which are familiar from Rosen's analysis [8].

Turning now to the consumer side of the market, let individual  $i$ 's tastes be represented by the strictly quasi-concave utility function  $v_i(x_i, q_i, z)$ ,  $i = 1, 2, \dots, n$ . Since it will be necessary in the analysis to treat consumer utility levels explicitly, suppose consumer  $i$  enjoys a utility level  $u_i$ , so that  $v_i(x_i, q_i, z) = u_i$ . Each consumer's income will equal his  $x$ -endowment  $w$  plus a  $1/n$  share of aggregate land rent  $r\bar{l}$  ( $\bar{l}$  is the fixed community land area). Letting  $R_i$  denote consumer  $i$ 's house rental payment, it follows that for the consumer to reach utility  $u_i$ ,  $R_i$  must satisfy

$$v_i(y(r) - R_i, q_i, z) = u_i, \quad (5)$$

where  $y(r) \equiv w + r\bar{l}/n$  represents income. Equation (5) implicitly defines the consumer's bid-rent function,

$$R_i = R_i(q_i, z, u_i; r), \quad (6)$$

which gives the rental payment consistent with the specified utility level as a function of house size, public good consumption, and land rent (see Rosen [8]). Differentiation of (5) shows that

$$\frac{\partial R_i}{\partial q_i} = \frac{v_{i2}(y(r) - R_i, q_i, z)}{v_{i1}(\quad)} > 0, \quad (7)$$

$$\frac{\partial R_i}{\partial z} = \frac{v_{i3}(y(r) - R_i, q_i, z)}{v_{i1}(\quad)} > 0. \quad (8)$$

The positive signs of (7) and (8) reflect the fact that rent must increase, causing a reduction in  $x_i$ , to hold utility constant when  $q_i$  or  $z$  increases. Note that the magnitude of the required rent increase depends on the MRS between  $q_i$  or  $z$  and  $x_i$ . In addition, it is easy to see from a diagram that the strict quasi-concavity of the utility function means that  $R_i$  is a strictly concave function of  $q_i$  and  $z$ .

Since migration is assumed to be costless, the utility levels of individuals with the same tastes must be uniform throughout the system of open communities. Furthermore, since the system is large, the utility level of any taste group cannot be influenced by the actions of housing producers or the local government in a single community. This fact means that consumer utility levels may be viewed as parametric in analysing producer and government behavior. As usual, housing producers will attempt to maximize profit in providing housing to the various consumers. The return that a producer can expect from renting a given house to a particular consumer is fixed by the parametric utility available to members of the consumer's taste group. If the producer offers a house of size  $q_i$  to individual  $i$  at a rent exceeding  $R_i(q_i, z, u_i; r)$  ( $u_i$  now denotes the parametric utility for  $i$ 's taste group), the consumer will be uninterested since better opportunities are available elsewhere in the community system. For given  $z$  and  $r$ , the maximum rent the producer can charge his customer without causing him to leave the community is  $R_i(q_i, z, u_i; r)$ . In view of this fact, the producer's optimization problem is easily characterized. Recalling that for given  $z$  and  $r$ , (4) defines a family of linear iso-profit rent contours in  $(x, q)$  space, it follows that the producer's goal is to choose the point on consumer  $i$ 's bid-rent curve  $R_i$ , which lies on the highest iso-profit line. The solution calls for a tangency (see Rosen [8]) between the (concave) bid-rent curve and an iso-profit line, as shown in Fig. 1 (asterisks indicate equilibrium values). The equilibrium conditions, which for fixed  $z$  and  $r$  yield equilibrium values for consumer  $i$ 's house size and producer profit from serving him ( $\pi_i$ ), are

$$R_i(q_i, z, u_i; r) = \pi_i + a(r)q_i + C(z, n)/n, \quad (9)$$

$$\frac{\partial R_i(q_i, z, u_i; r)}{\partial q_i} = a(r), \quad i = 1, 2, \dots, n. \quad (10)$$

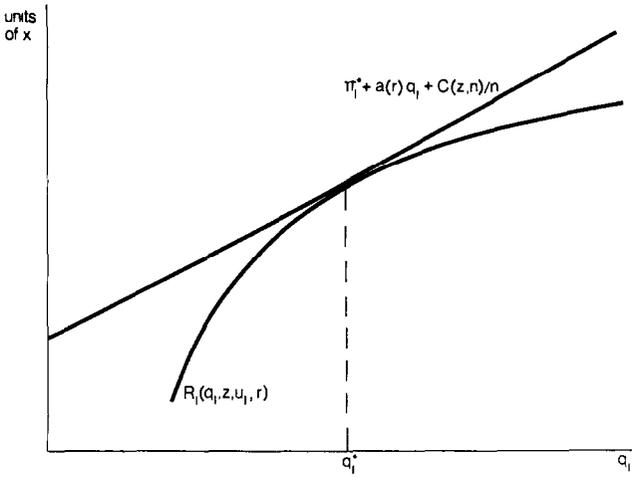


FIGURE 1

Equation (9) says that the bid-rent curve intersects the iso-profit line at  $q_i$ , while (10) indicates that the intersection involves a tangency.

To fully determine the housing market equilibrium for given  $z$  and  $u_i$ ,  $i = 1, 2, \dots, n$ , a land market clearing condition must be added to (9) and (10). Recalling the earlier expression for the magnitude of the land input, it follows that the land used in consumer  $i$ 's house is  $l_i = q_i/h(S(r))$ . Clearing of the land market requires  $\sum l_i = \bar{l}$  ( $\bar{l}$  is again the fixed community land area), and substitution yields the condition

$$\sum_{i=1}^n q_i = \bar{l}h(S(r)). \tag{11}$$

For given values of  $z$  and the  $u_i$ , (9)–(11) constitute  $2n + 1$  conditions to solve for the  $2n + 1$  unknowns  $r, q_i, \pi_i, i = 1, 2, \dots, n$ . The zero-profit requirement, which ultimately must be satisfied in equilibrium, will be introduced below.<sup>2</sup>

To explain how the local government chooses  $z$ , an expression for aggregate property value must first be derived. The value of a house is the price which the property will fetch on the open market once construction is complete. Since a buyer will be willing to offer at most an amount equal to the rent which the house commands minus the house tax liability, the value

<sup>2</sup>For simplicity, it is assumed that housing profits are paid to non-resident firm owners. Since profits must ultimately be zero in equilibrium, this assumption seems innocuous.

of the house inhabited by individual  $i$  is  $R_i(q_i, z, u_i; r) - C(z, n)/n$ .<sup>3</sup> Aggregate property value is then

$$\sum_{i=1}^n R_i(q_i, z, u_i; r) - C(z, n). \quad (12)$$

The fundamental behavioral assumption of the present analysis is that the local government chooses  $z$  to maximize (12) taking  $r$  and  $q_i$ ,  $i = 1, 2, \dots, n$ , as fixed (recall that the government is powerless to influence the  $u_i$ ). Thus, in appraising the desirability of an increase in  $z$ , the government focuses solely on the change in aggregate property value caused by capitalization of higher rents and higher house taxes for existing houses. It ignores the ultimate effect of its choice of  $z$  on the composition of the housing stock, on land rent, and on community population. Note that rents rise with an increase in  $z$  since higher public consumption makes every house more attractive, allowing producers to charge higher rents (reducing residents'  $x$  consumption) without driving utilities below prevailing external levels. While rents rise with  $z$ , the house tax increases as well, so that the value of a given house may either rise or fall.

The first-order condition for maximization of (12) is<sup>4</sup>

$$\sum_{i=1}^n \frac{\partial R_i(q_i, z, u_i; r)}{\partial z} = C_1(z, n). \quad (13)$$

Equation (13) completes the system of  $2n + 2$  equations which determines equilibrium values for the variables  $r$ ,  $z$ ,  $q_i$  and  $\pi_i$ ,  $i = 1, 2, \dots, n$ . Note that this system in fact describes the Nash equilibrium of a game in which the players are housing producers and the government. Producers view  $z$  as fixed in choosing the  $q_i$ , while the government takes the  $q_i$  as fixed in choosing  $z$ . Moreover, each agent assumes that land rent  $r$  is beyond his control and, of course, feels powerless to alter the parametric utilities.

The above conditions must be modified slightly to characterize an equilibrium where housing producer profits are zero. Setting  $\pi_i = 0$ ,  $i = 1, 2, \dots, n$ , in (9) would appear at first to be the proper modification, but it is immediately clear that the modified system (9)–(11) and (13) has  $2n + 2$

<sup>3</sup>Note that while the discussion so far has implicitly assumed that producers rent out their completed houses, so that no active market in houses actually exists, it is easy to see that analysis of an economy in which producers sell their completed properties to landlords who rent them to consumers is identical to the above. Note also that if the analysis had been carried out in a multiperiod model with durable houses, house value would equal the discounted present value of the difference between rent and taxes.

<sup>4</sup>The second-order condition is satisfied since the  $R_i$  are strictly concave in  $z$  and  $C(z, n)$  is convex in  $z$ .

equations, but only  $n + 2$  unknowns. However, if the utilities were free variables, replacing the  $\pi_i$  in the list of unknowns, then the modified system would solve for the  $2n + 2$  variables  $r, z, q_i$ , and  $u_i, i = 1, 2, \dots, n$ . What is the meaning of the resulting solution? A little reflection shows that the solution yields utility values which, when taken as parametric by housing producers and the local government, lead to a free-profit equilibrium where profits are identically zero. In other words, when the utilities assume the values implied by the modified system, optimizing behavior on the part of housing producers and the local government generates an internal equilibrium with zero profits. Since satisfaction of the zero-profit requirement is mandatory, it follows that the modified system ((9)–(11) and (13) with the  $\pi_i = 0$ ) is the relevant one for characterizing the equilibrium of the community. Note that this argument in a sense proceeds backward, taking the internal structure of the community as given and deducing what the external utility levels must be in order for that structure to be consistent with the surrounding environment.

It is easy to see that the implied zero-profit utilities for the various taste groups represented in the community population will depend on the overall size of the population and its taste-group makeup (for example, a high public good demander's zero-profit utility will depend on how many low demanders share his community). As a result, the prevailing parametric utilities consistent with zero profits in community *A* will in general be different from the prevailing utilities consistent with zero profits in community *B*. It should be clear, therefore, that while the population composition of a single community can be specified arbitrarily in the analysis, the composition of more than one community cannot be so specified without implying different prevailing utilities. Indeed, the population makeups of the communities in a system-wide equilibrium must be compatible in that a common set of prevailing utilities must be consistent with zero profits in each community. This point will become clearer in Section 4, where an example of a system-wide equilibrium is presented.

Before proceeding with the analysis, an important feature of the zero-profit equilibrium should be noted. Inspection of (9) shows that when  $\pi_i = 0$ , house value  $R_i - C/n$  is equal to  $a(r)q_i$ ; value is identically equal to production cost and thus is independent of  $z$ . This result highlights the fact that property value maximization has its effect during the transition to a zero-profit equilibrium. For given  $r$  and  $q_i$ , the influence of the public sector on individual house value disappears once equilibrium is reached. This feature of the model, which may at first appear peculiar, is similar to what happens in a standard profit maximization problem under constant returns to scale. In such a problem, the firm's objective function is insensitive in equilibrium to the level of output (profit is identically zero). Maximization of profit is nevertheless a well-defined goal.

It is useful to translate the zero-profit equilibrium conditions into more familiar terms, making use of (7) and (8). First, the variables  $u_i$  are eliminated from the problem, with the unknowns  $x_i$ ,  $i = 1, 2, \dots, n$ , appearing instead (recall that  $x_i = y(r) - R_i(q_i, z, u_i; r)$ ). Using (7), (10) then reduces to

$$\frac{v_{i2}(x_i, q_i, z)}{v_{i1}(x_i, q_i, z)} = a(r), \quad i = 1, 2, \dots, n, \quad (14)$$

which states that each consumer's marginal rate of substitution between housing and the numeraire equals the marginal cost of housing. Using (8), (13) becomes

$$\sum_{i=1}^n \frac{v_{i3}(x_i, q_i, z)}{v_{i1}(x_i, q_i, z)} = C_1(z, n). \quad (15)$$

Equation (15) is, of course, the well-known Samuelson condition, which states that the sum of the marginal rates of substitution between the public good and the numeraire equals the marginal cost of the public good. It is this condition which emerges from property value maximization by the local government. While (11) needs no simplification, substitution for the bid-rent function in (9) (with  $\pi_i = 0$ ) gives

$$w + r\bar{l}/n = x_i + a(r)q_i + C(z, n)/n, \quad i = 1, 2, \dots, n \quad (16)$$

(recall  $w + r\bar{l}/n \equiv y(r)$ ). Equations (14)–(16) and (11) determine equilibrium values for  $r$ ,  $z$ ,  $q_i$ , and  $x_i$ ,  $i = 1, 2, \dots, n$ . It is easily seen that these equations characterize a Pareto-efficient allocation within the given community. The Lagrangean for the Pareto problem is

$$\begin{aligned} v_1(x_1, q_1, z_1) - \sum_{i=2}^n \delta_i (v_i(x_i, q_i, z) - \bar{u}_i) \\ - \gamma (nw - \sum x_i - \sum x_i^h - C(z, n)) \\ - \sum \epsilon_i (q_i - H(x_i^h, l_i)) - \xi (\sum l_i - \bar{l}), \end{aligned} \quad (17)$$

where  $x_i^h$  denotes the  $x$  input into consumer  $i$ 's house and the  $\bar{u}_i$ ,  $i = 2, 3, \dots, n$ , are fixed utility levels. Note that the third-to-last restriction is the aggregate  $x$  constraint, while the subsequent restrictions are housing output constraints and the aggregate land constraint. Noting (2) and (3), it is straightforward to verify that the internal equilibrium characterized by (14)–(16) and (11) satisfies the necessary conditions for Pareto efficiency

found by differentiating (17). Thus, the internal equilibrium of the community will be Pareto-efficient. Moreover, since the community was arbitrarily chosen, it follows that *all* communities will be internally efficient in a system-wide equilibrium generated by property value maximization.

It is important to realize that the Nash nature of the local government's property-value-maximizing behavior is responsible for the efficiency of internal equilibrium. If the government were to take account of the influence of its output choice on the  $q_i$  and  $r$  in maximizing (12), then the Samuelson condition would not emerge and the equilibrium would be inefficient. This suggests the important conclusion that government search for an actual property value maximum will not lead to a Pareto-optimum.<sup>5</sup>

### 3. THE EFFECT OF A PROPERTY TAX

Since the property tax is in reality the main source of revenue for local governments, it is important to ascertain how the introduction of such a tax changes the preceding results. Letting  $F$  again denote rent, house value  $V$  is determined under property taxation by the relationship  $V = F - \tau V$ , where  $\tau$  is the property tax rate ( $\tau V$  is the property tax liability). Solving for  $V$  yields  $V = F/(1 + \tau)$ , and the fixed profit condition  $F - \tau F/(1 + \tau) - a(r)q = \pi$  implies that the rental payment required for profit level  $\pi$  is

$$F = (1 + \tau)(a(r)q + \pi). \quad (18)$$

Using (18), a zero-profit equilibrium requires satisfaction of

$$R_i(q_i, z, u_i; r) = (1 + \tau)a(r)q_i, \quad (19)$$

$$\frac{\partial R_i(q_i, z, u_i; r)}{\partial q_i} = (1 + \tau)a(r), \quad i = 1, 2, \dots, n, \quad (20)$$

together with the land market clearing condition (11). The government's budget constraint, which relates  $z$  and  $\tau$  via aggregate property value  $\sum R_i/(1 + \tau)$ , is

$$\frac{\tau}{1 + \tau} \sum_{i=1}^n R_i(q_i, z, u_i; r) = C(z, n). \quad (21)$$

While aggregate value could be maximized by choice of  $z$  and  $\tau$  subject to the constraint (21), a simpler approach is to eliminate  $\tau$  from the aggregate

<sup>5</sup>In this context, it is interesting to note that analysis by Starrett [10] shows that land rent changes in a spatial setting generally fail to measure properly the benefits of a public improvement. Thus, when general equilibrium feedbacks are considered, land value changes (changes in the difference between land rent and public sector costs) are invalid as a guide for choosing optimal public outputs

value expression using the relationship  $V_i = R_i - \tau V_i$ . Summing over  $i$  yields  $\Sigma V_i = \Sigma R_i - \tau \Sigma V_i = \Sigma R_i - C(z, n)$ , noting that the government's budget constraint may be written  $\tau \Sigma V_i = C(z, n)$ . Thus, aggregate value is again equal to (12), so that (13) is the first-order condition for property value maximization. The equilibrium values for the variables  $z, \tau, r, q_i, u_i, i = 1, 2, \dots, n$ , are therefore determined by (19), (20), (11), (21), and (13). Although property value maximization once again yields the Samuelson condition via (13), the community equilibrium is not Pareto-efficient due to the distortion introduced by the property tax. From (20), the marginal rate of substitution between housing and the numeraire is not equal to the marginal cost of housing in equilibrium, as required by efficiency, but instead equals  $(1 + \tau)$  times marginal cost. In spite of this distortion, a limited efficiency result may be stated. In particular, it is easy to see that the internal equilibrium defined by the above equations is Pareto-efficient *conditional on the (inefficient) equilibrium housing stock*. That is, if the  $q_i$  are fixed at their equilibrium values and a Pareto optimum is characterized (see (17)), the necessary conditions will be fulfilled by the above equilibrium conditions. Thus, while consumption of  $z$  and the numeraire is nonoptimal in general, consumption levels are efficient taking as given the (non-optimal) housing stock.

#### 4. GLOBAL EFFICIENCY

The analysis in Section 2 established that under a house tax regime, property value maximization by local governments generates a system-wide equilibrium in which communities are internally Pareto-efficient. An important question, however, is whether such an equilibrium is efficient in a global sense. That is, might there exist some reallocation of individuals among communities which leads to a Pareto-superior outcome? This section of the paper will show that the answer to this question is affirmative; a simple example will be constructed of a globally inefficient system-wide equilibrium in which each community is internally Pareto-efficient. This example establishes that property value maximization need not guide the economy to a global optimum. Also, the discussion clarifies the nature of a system-wide equilibrium.

Imagine that the economy has two taste groups, and suppose that if individual  $i$  belongs to taste group  $k$ , his utility function is  $x_i + b(q_i) + m_k(z)$ ,  $i = 1, 2, \dots, n$ ,  $k = 1, 2$ . Note that tastes differ only in the function  $m_k$ . Let  $n$  as before equal community population and let  $\theta$  equal the proportion of the population composed of type 1 individuals. It may be shown that the zero-profit equilibrium solutions for land rent and the private and public consumption levels of the taste groups depend only on the parameters  $\theta$  and  $n$  (land area is assumed to be identical for all communities). Substituting the equilibrium solutions back into the two

utility functions, the zero-profit utilities of the two taste groups may also be expressed simply as functions of  $\theta$  and  $n$ . Let  $f_k(\theta, n)$ ,  $k = 1, 2$ , indicate these utilities. It may be shown that  $\partial f_1/\partial\theta > 0$  and  $\partial f_2/\partial\theta < 0$ .<sup>6</sup> These inequalities state that for a given  $n$ , the type-one utility level is an increasing function of the proportion of type-ones in the community, while the type-two utility is a decreasing function of this proportion, conclusions which are intuitively reasonable.

With this background, an example of a globally inefficient system-wide equilibrium can be constructed easily. Suppose the total size of each taste group equals  $mP$ , where  $m$  and  $P$  are both large integers, and let each group be divided into  $m$  equal subgroups of size  $P$ . Then form  $m$  communities by merging subgroups, so that each community has a population of  $2P$  composed of  $P$  type-ones and  $P$  type-twos. The zero-profit utilities for the groups in each community are  $f_k(1/2, 2P)$ ,  $k = 1, 2$ . Now as should be clear from the earlier discussion, if each community faces parametric taste group utilities of  $f_k(1/2, 2P)$ ,  $k = 1, 2$ , then optimization by housing producers and the local government will yield identical internal equilibria with zero profits. Therefore, the given population configuration constitutes a community system equilibrium when the prevailing utilities equal  $f_k(1/2, 2P)$ ,  $k = 1, 2$ .

Since the equilibrium was generated by property value maximization, it follows from above that each of the  $m$  communities is internally Pareto-effi-

<sup>6</sup>Since the function  $b$  is the same for both taste groups, condition (14) gives identical solutions for  $q$  for the groups:

$$b'(q) = a(r). \quad (14')$$

As a result, the land market clearing condition (11) becomes

$$nq = h(S(r)) \quad (11')$$

Together, (14') and (11') determine  $q$  and  $r$  as functions of  $n$ . The Samuelson condition (15) may be written

$$n\theta m'_1(z) + n(1 - \theta)m'_2(z) = C_1(z, n), \quad (15')$$

which determines  $z$  as a function of  $\theta$  and  $n$ . Finally,  $x$ , which is the same for both groups, is given by (16):

$$x = w + r\bar{l}/n - a(r)q - C(z, n)/n. \quad (16')$$

Given the previous solutions for  $q$ ,  $r$ , and  $z$ , it follows that  $x$  is a function of  $\theta$  and  $n$ . Substituting the above solutions back into the utility functions of the two groups yields  $f_1(\theta, n)$  and  $f_2(\theta, n)$ . The signs of  $\partial f_k/\partial\theta$ ,  $k = 1, 2$ , are established by a straightforward calculation

cient. However, could some arrangement of the population lead to a Pareto-superior outcome? It is easy to see that the answer to this question is affirmative. Consider rearranging the population into  $m/2$  homogeneous type-one communities of size  $2P$  and  $m/2$  homogeneous type-two communities of size  $2P$ . Consider the allocations corresponding to the zero-profit equilibria in these communities. The zero-profit utility of type-ones in a homogeneous community of size  $2P$  is  $f_1(1, 2P)$ , while the corresponding utility for type-twos is  $f_2(0, 2P)$ . Recalling that  $f_1$  and  $f_2$  are respectively increasing and decreasing in  $\theta$ , it follows that  $f_1(1, 2P) > f_1(1/2, 2P)$  and  $f_2(0, 2P) > f_2(1/2, 2P)$ . This means that an allocation other than that afforded by the system-wide equilibrium yields higher utility levels for both groups. Hence the system-wide equilibrium is globally inefficient.<sup>7</sup> This result shows that while the Nash-type property value maximization analyzed in this paper guarantees internal Pareto-efficiency, it may fail to generate an optimal assignment of consumers to communities. Since value-maximizing governments view community populations as parametric, this failure is understandable.

## 6. CONCLUSION

The purpose of this paper has been to show in the context of a carefully constructed model how property value maximization can generate public sector efficiency. It was established that under a house tax regime, Nash-type property value maximization (in which the government ignores its indirect influence on the local economy) leads to a system-wide equilibrium in which communities are internally Pareto-efficient. It was shown, however, that the equilibrium may be inefficient in a global sense. The possibility of global inefficiency means that the present results are considerably weaker than those of Sonstelie and Portney, who argue that profit maximization by local governments generates global optimality. In view of the greater precision of the present analysis, it appears that the weaker efficiency result is more credible.

Future research could be directed toward deriving a more comprehensive government behavioral code which would preclude the existence of globally inefficient equilibria. Since the possibility of global inefficiency in the present model can be traced to the parametric-population assumption, a globally efficient behavioral code would clearly require government control of community populations through a mechanism such as zoning or restriction of migration. While fiscal zoning has been discussed in the literature

<sup>7</sup>It is important to realize that while the homogeneous community configuration is superior to the heterogeneous one, it does not necessarily represent the global optimum. Further reshuffling of community populations might increase consumer welfare even more (for example, homogeneous communities of size  $4P$  might be superior)

(see Hamilton [6]), deriving a zoning decision rule for local governments capable of generating global efficiency in a model like the present one would seem to be a very difficult task. Further work in this direction might, however, produce important results, providing an ultimate vindication of the Tiebout approach.

Finally, it is natural to ask whether the type of property value maximization analyzed in this paper constitutes an operational method for achieving (internal) Pareto-efficiency. The answer to this question appears to be a qualified yes. Since in reality the changes in a community's housing stock and land rent in response to an increase in its public good level will be slow compared to the capitalization of higher house rents and house taxes for existing structures, a local government might be able to engage in the kind of behavior required to deduce whether or not its public output is efficient. Of course, to make this claim rigorous, the analysis would have to be redone using an intertemporal model which recognizes the longevity of houses.

In conclusion, it should be pointed out that the analysis in this paper has empirical significance. The question of whether local public outputs actually maximize aggregate property values (holding community housing stocks fixed) can be addressed by computing a cross-section regression relating aggregate (or median) values to public spending and other explanatory variables. The regression results indicate whether or not communities are internally Pareto-efficient. See Brueckner [2, 4] for details of the argument and empirical results.

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