RAND Journal of Economics
Vol. 27, No. 2, Summer 1996
pp. 281–309

Product variety and firm agglomeration

Jeffrey H. Fischer*

and

Joseph E. Harrington, Jr.**

For the purpose of explaining interindustry variation in the geographic distribution of firms, we explore the impact of product heterogeneity on the incentives for firms to cluster in the presence of a ubiquitous "periphery" of stand-alone firms. Our analysis revolves around two counteracting forces. Greater product heterogeneity increases consumer search, which raises the amount of shopping at a cluster. Since this results in greater demand for a firm that joins the cluster, this effect increases the incentive to cluster. However, greater product heterogeneity gives stand-alone firms more local monopoly power. Since this raises their price-cost margins, this effect increases the incentive for a firm to stand alone. Our analysis shows that the former effect typically dominates, so that greater firm agglomeration is associated with industries characterized by greater product heterogeneity.

1. Introduction

■ Observation suggests that the clustering of firms selling similar products is an important phenomenon. Shopping malls house multiple clothing and shoe stores; antique dealers and jewelry dealers are often found in areas colloquially named "Antique Row" or "Jewelers' Row"; and even car dealers tend to locate close to one another, to the point where these agglomerations are marketed as "auto malls." At the same time, such agglomeration is by no means all-pervasive. Even industries that tend to agglomerate support firms that locate away from clusters.

This article investigates the source of interindustry variation in agglomeration by firms. We begin by providing some evidence on clustering by firms. This evidence documents considerable interindustry variation in agglomeration and suggests a possible factor to systematically explain this variation—industries with more heterogeneous products appear to engage in more clustering. The main contribution of the article is in developing a theory based upon consumer search that makes predictive statements about the relationship between product heterogeneity and firm agglomeration. A key distinction between our theory and preceding work is that we seek to take account of

* Federal Trade Commission.

** Johns Hopkins University.

The authors would like to thank Bruce Hamilton and an anonymous referee for very helpful comments. The views expressed in this article are those of the authors, and are not necessarily the views of the Federal Trade Commission or any individual commissioner.

a significant empirical fact about clustering: clusters of firms are generally coincident with a ubiquitous periphery of unclustered firms. Using this framework, we determine conditions under which a cluster will exist and parameter changes that increase the size of the cluster (if one exists) or increase the likelihood a cluster will exist. We find that greater product heterogeneity makes it more likely that a cluster is viable. However, since increased heterogeneity is found to increase the size of the periphery as well as that of the cluster, the extent of clustering need not be strictly monotonic in the degree of product heterogeneity. In spite of the lack of a strictly monotonic relationship, our analysis shows there to be a general tendency for clustering to be greater in markets with more heterogeneous products.

□ **Some empirical observations.** Using data from the Baltimore metropolitan area, we examined the relative spatial concentration of firms across nine markets.[1] The index of spatial concentration is the distance between each firm and its closest n competitors, where n is allowed to range from 2 to 5. Since some markets will naturally have more firms, because of greater demand or smaller minimum efficient scale, and more firms in a fixed geographic area will reduce the mean distance between competitors, one must control for the number of firms in a market when making intermarket comparisons. We then normalize the mean distance between firms by the number of firms in that market. Summary statistics of all nine markets are presented in Table 1. The markets for shoe stores and antiques are by far the most concentrated, with computer stores and auto dealers third and fourth using the nearest-five-firm measure of agglomeration. The other three measures of agglomeration provide similar results, although with some variation in the rankings. This is because almost all shoe stores were located in malls or in one of three nonmall clusters, while over half the antique dealers located in a four-block area. Auto dealers, as a rule, were similarly located in clusters. At the other extreme, supermarkets and movie theaters were consistently the least-concentrated markets, regardless of the measure of concentration. The conclusion we draw is that our ranking is robust to which of these four measures we use.

The underlying observations for shoe stores and video stores are shown in Figures 1 and 2. The geographic area for each market is a rectangle approximately 3½ miles by 2 miles, comprising most of Baltimore City. Circled numbers represent the number of stores in a single location, such as a mall or strip shopping center. The figures provide a visual representation of Table 1. In Figure 1, most shoe stores are located near other shoe stores, though some stand-alone stores remain. In contrast, in Figure 2, video stores are seen to be widely dispersed throughout the area.

The markets with the greatest degree of agglomeration are the ones that appear to exhibit the greatest degree of product differentiation and require the largest amount of consumer search. Antiques are often one-of-a-kind objects and are always difficult to compare in terms of characteristics. In addition, the lack of a standardized product makes price comparisons meaningless without inspecting the products. These factors encourage repeated search and comparison shopping. The sale racks of women's shoes display a similarly high degree of heterogeneity, with dozens of manufacturers producing a multiplicity of styles. Which store carries an acceptable style in the correct size in a suitable color is unknown to the shopper ahead of time because one can determine an acceptable style only through visual inspection of the product. It is the necessity of visually and/or physically inspecting the product that links the heterogeneous search goods.

Cars, too, share this characteristic. While there are fewer brands of cars than there are styles and sizes of shoes, buying a car is such a major purchase that consumers are

[1] Details of the empirical results may be found in Fischer (1992).

TABLE 1 Mean Distance Across Markets

Market	Number of Firms	Adjusted D_5^j	Adjusted D_4^j	Adjusted D_3^j	Adjusted D_2^j
Shoes	89	25.08	17.74	11.38	5.58
Antiques	61	26.32	17.95	10.88	5.66
Computers	40	48.00	42.61	35.89	27.35
Automobiles	30	51.81	39.62	29.33	20.72
Clinics	30	63.10	54.45	46.92	41.81
Gas stations	72	63.57	55.51	46.90	36.83
Video stores	55	63.94	56.84	49.22	41.94
Supermarkets	25	78.16	70.90	63.47	56.50
Theaters	14	83.39	73.96	67.26	62.94

Sources: C & P Telephone Yellow Pages (1990) and ADC Baltimore and Baltimore County Street Map. D_i^j is the mean distance of each firm in market j from its nearest i rivals multiplied by the square root of the number of firms in market j. This distance is measured in units of 2,000 feet (as each inch of the map equalled 2,000 feet).

FIGURE 1

SHOE STORES

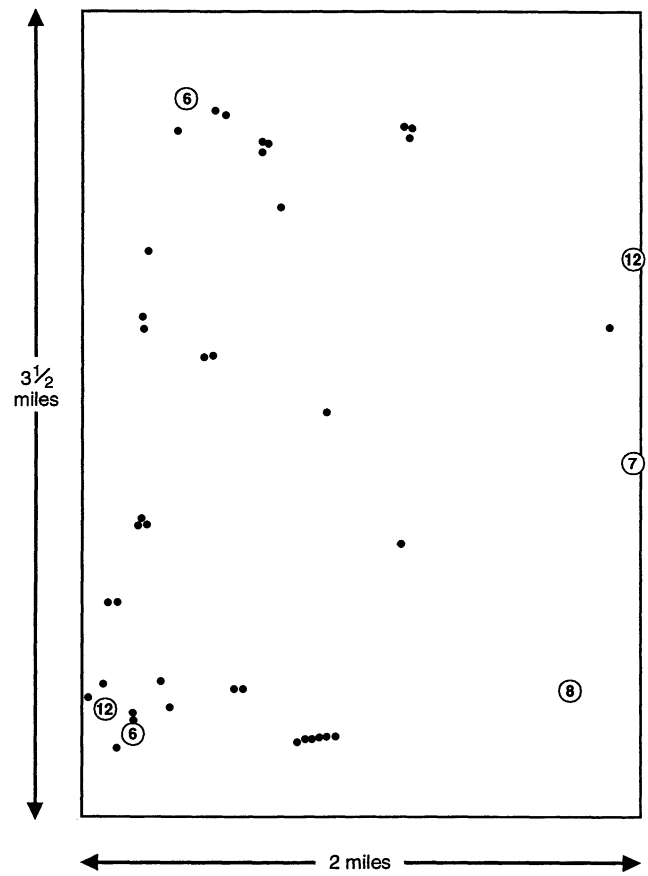
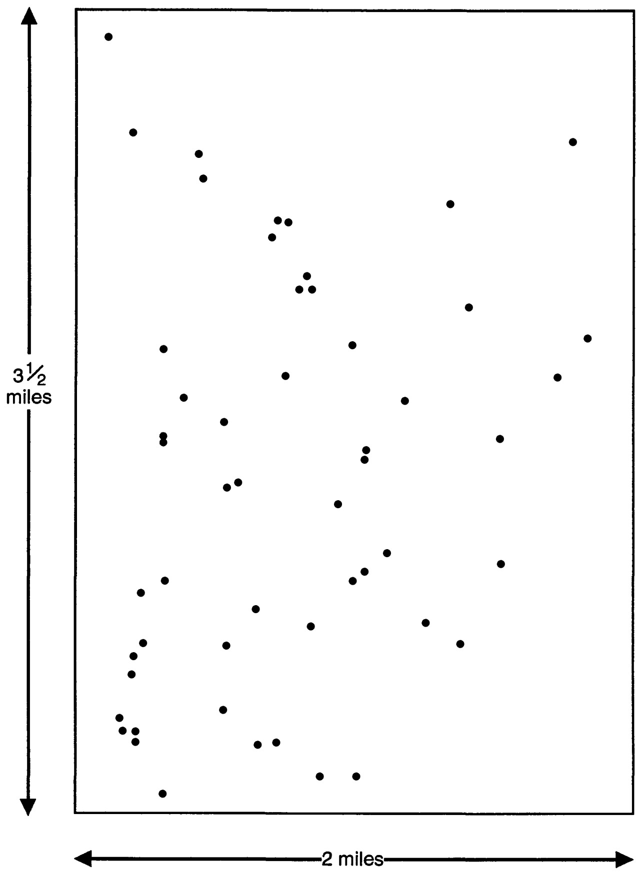


FIGURE 2

VIDEO STORES



willing to spend a great deal of time and effort to find the right model. To do so requires the same on-site inspection of the product as do shoe and antique shopping. At the same time, dealers, knowing the incentives consumers have to engage in comparison shopping, work to differentiate their products from those of the competition, reinforcing the incentives consumers have to physically inspect the cars in order to determine which characteristics are important and truly different across product lines.

The other markets are ones in which there is either little product differentiation (video stores, gas stations, supermarkets) or little scope for comparison shopping (theaters, whose products are observable from the newspaper, and clinics). Regardless of the index used, video stores and gas stations consistently display a large degree of geographical dispersion, and these are stores with highly homogeneous products. In the case of video stores, most stores carry roughly the same products (stores specializing in certain titles, such as foreign movies or pornographic films, could be considered separate markets; within these markets the same argument applies), so there is little point in sampling more than one store. Similarly, gas stations have little to differentiate themselves from one another, limiting the amount of search a consumer would be willing to undertake. Supermarkets vary in brands carried, price, quality, and quantity of goods, but essentially carry the same types of goods, with a great deal of overlap among firms. A quart of milk from one dairy is distinguishable among substitutes only by price. Hospitals and clinics, though differing greatly in terms of quality in providing care for major concerns, are roughly equivalent for minor events such as stitching small wounds, and one is rarely tempted to search for better choices in emergencies. Theaters differ slightly in amenities, but the main product—the film—is a perfect substitute for the product of theaters carrying the same film and is a highly imperfect substitute for all other films. Once a moviegoer determines the film he or she wants to see, there is little point in travelling beyond the nearest theater to do so. While these numbers are not the result of an exhaustive survey of clustering behavior, they do suggest that the degree of clustering is positively related to the degree of product differentiation.

It should be noted that the degree of heterogeneity is not the only characteristic by which these markets differ. Two other characteristics that come to mind are the size of the average purchase and the frequency with which consumers purchase the product. For both more heterogeneous products and larger-purchase products, search is more valuable to the consumer. For products purchased frequently, on the other hand, such as groceries, search may be less valuable because consumers have an accumulation of information about product prices and characteristics. Automobiles, for example, are purchased infrequently and at a high average price, reinforcing the tendency to search that product heterogeneity provides. Videos are often frequent purchases at a low price, reinforcing the tendency not to search.

Despite the apparent correlation between heterogeneity and agglomeration, not all firms in a market choose to cluster. Even markets with a high proportion of clustered firms also contain unclustered firms. It is important to understand firms' incentives to remain isolated at the same time that other firms choose to cluster.

□ **Objective of present research.** There are two facts that we draw from this evidence. First, there is significant interindustry variation in the degree of agglomeration. Second, products that are more heterogeneous appear to result in greater agglomeration.

Some casual reasoning suggests that firms should indeed find it more profitable to cluster when products are more heterogeneous. With greater variation in the products being offered, consumers can anticipate doing more search in order to find the best product-price combination. Greater anticipated search makes going to a cluster more attractive, as a consumer can then search several firms at once. This results in more consumers visiting the cluster and cluster firms having more demand for their products. In other words, greater product heterogeneity magnifies the main benefit from joining a cluster, which is that it tends to attract consumers. It also tends to reduce the main cost from joining a cluster, which is that close proximity of firms intensifies price competition. Price competition will generally be less, the more differentiated firms' products are. These two forces suggest that the more heterogeneous firms' products are, the greater the profit from joining a cluster and thus the more agglomeration we should observe.

Unfortunately, further reflection shows that matters are not so simple. This discussion ignores the presence of unclustered, or periphery, firms. We have argued that as product heterogeneity rises, the price that a firm can charge at the cluster would rise, and this partially contributes to the rise in profit from joining a cluster. However, the price that a periphery firm can charge should also rise as products become more differentiated. Whether or not more consumers go to the cluster depends not only on the expected intensity of search but also on the price of the product at a cluster firm relative to its price at a periphery firm. If in response to a rise in product heterogeneity the cluster price rises sufficiently faster than the periphery price, fewer consumers might go to the cluster. In that case, whether there is more clustering for a more heterogeneous product depends on whether the higher cluster price sufficiently compensates for weaker demand. Indeed, it is not unreasonable to expect the cluster price to rise faster than the periphery price in response to a rise in product heterogeneity. The effect of relieving competition through greater product heterogeneity might be expected to be of greater significance where competition is greater. Since periphery firms have some local monopoly power while cluster firms do not, the effect would seem to be greater at the cluster. In any event, the exact effect of increased product heterogeneity on the profitability of clustering is no longer so clear.

What this discussion reveals to us is the need to develop a formal model that encompasses these various effects and can be used to assess their relative magnitudes. This is what we do in this article. We develop a model in which firms have the opportunity to agglomerate in the presence of a ubiquitous periphery of firms. Each firm decides whether or not to join the cluster or the periphery. Once located, firms decide on price. Consumers optimally search the cluster (if one exists) and the periphery. We characterize a free-entry (subgame-perfect) equilibrium. Our results show that because of the counteracting effects described above, there is not a strictly monotonic relationship between product heterogeneity and some measures of firm agglomeration. However, our results reveal a general tendency: we find there to be greater clustering in markets with more heterogeneous products. Specifically, we find that a firm's profit from locating at the cluster is strictly increasing in the degree of product heterogeneity. It follows that the greater the product heterogeneity, the greater the likelihood of a cluster in equilibrium and, if there is a cluster, the more firms will locate there. But even if product heterogeneity increases the number of cluster firms, it need not imply a greater degree of agglomeration, as the number of periphery firms may be rising even faster. To take account of this factor requires assessing the impact of product heterogeneity on the relative profitability of joining the cluster and going it alone in the periphery. This analysis demands the use of numerical simulations. Our measure of agglomeration is the proportion of all firms that locate in the cluster. Because of counteracting forces, we do not find agglomeration to be strictly increasing in product heterogeneity. However, there is a clear and strong tendency in the simulations, and it is for there to be greater agglomeration when products are more heterogeneous.

□ **Related work.** The phenomenon of firm agglomeration has been examined in the theoretical work of Eaton and Lipsey (1979), Stahl (1982), Wolinsky (1983), and Dudey (1990). This work differs from ours in two respects. First, the objective of these articles was to derive conditions for a cluster to exist in equilibrium. While we do this as well, our primary objective is to understand the role of product heterogeneity in explaining interindustry variations in firm agglomeration. Second, as described below, our model of clustering differs in a number of respects from the models previously developed in the literature. Though our model has its own weaknesses, we believe it is the most appropriate one for assessing the impact of product heterogeneity on clustering.

Being the first to venture into the phenomenon of clustering, it is not surprising that Eaton and Lipsey (1979) and Stahl (1982) should have some restrictive assumptions. In particular, there are two central assumptions that make their models unsuitable for our objectives. First, they assume that consumers conduct a fixed number of searches: in Stahl it is one search and in Eaton and Lipsey it is two. Since the way that the pattern of consumer search responds to the presence of a cluster is central to our analysis, this specification assumes away a potentially important effect. Second, they assume that price is exogenous. This is clearly unsatisfactory, as it assumes away the central cost to joining the cluster, which is that price competition is more intense than if one located apart from other firms.

Neither Wolinsky (1983) nor Dudey (1990) is subject to these criticisms, in that both allow for optimal consumer behavior and endogenize price as well as location decisions. In contrast to our objectives, Dudey focuses exclusively on consumer search over price, since firms are assumed to sell homogeneous products (and engage in Cournot competition). His analysis yields the finding that all clusters must have an identical number of firms. This prediction is inconsistent with the empirical observation that some firms join a cluster and others do not. On the other hand, a nice feature of Dudey is that it allows for multiple clusters, whereas we restrict attention to an equilibrium with a single cluster.

The model of Wolinsky (1983) is closest to ours, and in fact our model should be seen as building upon it in that we maintain his motivation for search, which is that consumers are uncertain as to the characteristics of firms' products. The central contribution of Wolinsky is the derivation of conditions such that an equilibrium exists in which all firms cluster. In contrast, we consider clustering in the presence of many isolated firms. That is, we establish the existence of an equilibrium in which some firms cluster but many more locate away from the cluster. We believe this distinction to be important. It is rarely if ever observed that all firms cluster together. Typically, clustering takes place in an environment in which there are many isolated firms. Furthermore, our initial discussion suggests that this property may be important with regard to understanding the observed extent of agglomeration. In assessing the influence of product heterogeneity, one must take into account how it affects the cluster price relative to the periphery price and how these relative prices (as well as the degree of product heterogeneity) affect a consumer's search pattern. Finally, since Wolinsky shows when an equilibrium exists in which all firms cluster, his theory tells us only when we should expect to observe a cluster, not when we should *not* expect to observe a cluster. In comparison, we show the general existence of an equilibrium for our model and establish conditions whereby a cluster is present and when one is not present (in which case all firms isolate themselves from one another). There are, however, several advantages to Wolinsky's model. First, he models firms as locating in product (as well as geographic) space, while we assume firms are endowed with their products. Second, his model is rooted in a spatial setting, while our model has no apparent spatial analogue. In spite of its apparent conflict with physical reality, we hope to convince the reader that our model is a plausible representation of a consumer's perception of reality.

2. The model

■ In this model each firm sells a single heterogeneous product. Consumers, endowed with preferences over the varieties of this product, know the space of possible varieties and the location of firms in geographic space, and they have expectations of prices at each location but do not know the characteristics of the product sold by a specific firm. Thus the purpose of search is to evaluate firms' products. In this section we describe consumers and firms, as well as the set of possible actions for each agent, and characterize equilibrium.

□ **Firms.** A large number of firms sell a heterogeneous product in one of two types of locations: a cluster, in which firms are close to one another geographically, or a "periphery," in which firms are spread out over the market area and isolated from one another. Denote the number of periphery firms as N and the number of cluster firms as n. We allow for the possibility that $n = 0$, but think of N as being large. The geographical aspect of both the cluster and the periphery is not meant to be taken too literally. The cluster is merely an approximation to a location in which firms are located close to one another, such as shopping malls, strip shopping centers, garment districts, music districts, auto malls, and so on. Such an agglomeration will be close to some consumers and distant from others. The periphery is an approximation for localized stores—neighborhood shoe stores, corner drug stores—that, for one reason or another, choose not to be part of the cluster.

Each firm sells at zero unit cost a variety drawn randomly from the set of possible varieties. Firms do not choose varieties, but are assigned them exogenously by a draw from a uniform distribution. Firms enter by incurring the cost of entry of $K > 0$. Firms enter if and only if expected profits are nonnegative. At the time of entry, a firm decides whether to locate in the periphery or at the cluster. Once set in its location, firm i then chooses a price p_i to maximize profits π_i^J:

$$\pi_i^J(p_i; \mathcal{P}, n, N) = p_i D_i^J(p_i; \mathcal{P}, n, N), \tag{1}$$

where $J \in \{C, P\}$, $\mathcal{P}$ is the vector of all firms' prices, and $D_i^J(p_i, \mathcal{P}, n, N)$ is the (expected) demand for firm i's variety in location J at price p_i when n firms are located in the cluster, N firms are located in the periphery, and firms charge $\mathcal{P}$. Firms maximize (1) with the beliefs (a) that other firms will charge $\mathcal{P}$, (b) all firms have varieties that are uniform identically and independently distributed draws from the set of varieties, and (c) firms' locations are fixed and common knowledge. We derive D_i^J below.

□ **Consumers.** Consumers differ from one another in two respects: the ranking of the varieties of the heterogeneous product and the cost of searching the cluster. Each firm i sells a single variety that yields a valuation to consumer l of v_i^l, which is the consumer's maximum willingness to pay for variety i. v_i^l is drawn from a uniform distribution $F(v)$, defined over the interval $[\underline{v}, \overline{v}]$.[2] Thus varieties may be thought of as random draws from some underlying space of varieties; the uncertainty of the valuations represents both the firm's uncertainty about the perception of an individual consumer to its product and the uncertainty that the consumer himself has about his valuation of a particular firm's product before actually arriving at the store and examining the variety. $h \equiv (\overline{v} - \underline{v})$ will measure the degree of product heterogeneity, since it equals the range of possible valuations a consumer might assign to firms' products.

The cost to consumer l of searching a firm in the periphery is $t > 0$ and of searching the cluster is c^l. t is common across consumers, while c^l is drawn from $[\underline{c}, \overline{c}]$ according to a distribution $G(c)$. Once at the cluster, a consumer may search all firms there costlessly. Varying c^l captures the idea that some consumers are close to the cluster while some are far away; a constant t reflects relatively uniform opportunities for visiting periphery stores regardless of the consumer's address.

An important assumption we make concerns the beliefs of consumers. We assume that consumers correctly perceive the number of firms at the cluster, n, but view the periphery as containing an unlimited number of firms. More to the point, consumers are assumed to view the periphery as being inexhaustible in that there are more firms than a consumer would ever anticipate searching. Though this perception is technically incorrect, as N will be finite in equilibrium, we see our model as being applicable to markets where N is quite large, so that this assumption is a good approximation to reality (and actually may be a more accurate representation of consumers' perception of reality). Such an assumption would clearly be inappropriate with respect to the cluster, as it corresponds to a small geographic area with typically a small number of firms and low intracluster search costs so that consumers will often search all firms at the cluster.[3]

[2] Wolinsky (1983) also employs the assumption of a uniform distribution of preferences.

[3] That consumers would perceive the periphery as containing an unlimited number of firms when N is large (but finite) does have an anthropological basis. The numerical system of the Bushmen of Botswana

Consumers search with recall, and always have the option of stopping search, continuing search by sampling a firm in the periphery, or continuing search by visiting the cluster. If the consumer chooses not to search any further, he has the choice of buying from the firm yielding the highest utility obtained thus far, or opting out of the market, yielding a utility of zero. Allowing recall in the periphery, with a large number of firms to potentially search, is innocuous; Kohn and Shavell (1974) show that the privilege of recall will never be used. In the cluster, however, the consumer may costlessly return to a previously sampled firm and hence may invoke recall. The payoff to consumer l from buying the variety at firm i at a price p_i is $v_i^l - p_i - C$, where C is the total search cost incurred. There is assumed to be a continuum of consumers represented by $[0, \mathcal{L}]$. Thus $\mathcal{L}$ is a measure of the size of the market.

To review, the key assumptions on search costs are as follows. First, intracluster search costs are small relative to intraperiphery search costs. This we achieve by setting intracluster search costs to zero while assuming that each periphery search entails a positive cost t. Second, because of the uniqueness of the cluster and the ubiquitousness of the periphery, all consumers are assumed to face the same costs of searching the periphery but different costs of searching the cluster. Third, the cost of searching a periphery firm is independent of the number of periphery firms. This last assumption is less compelling than the other two. If the periphery is a fixed geographic space, then more periphery firms should shorten the average geographic distance between a consumer and a periphery firm, thereby lowering (travel) search costs. However, it is an assumption that tremendously simplifies the analysis and, for at least some ranges of parameter values, is not too gross a violation of reality.

□ **Equilibrium.** To summarize, the game is played in three stages. Firms enter and decide on location in the first stage. They choose price in the second stage, given the location decisions of all firms, summarized by (n, N). Finally, consumers choose an initial location to search and a search strategy, given the location of firms and conjectures on the price of each firm.

The solution concept used is subgame-perfect equilibrium. Because of the symmetry of firms, we focus on symmetric equilibria, in which all cluster firms charge a common price p^C and all periphery firms charge a common price p^P. A solution is a number of firms in each location, N^* and n^*, a profile of price strategies for the $N^* + n^*$ firms, and a profile of search strategies for the $\mathcal{L}$ consumers.

We will need to make several parametric assumptions. First, the minimum cost for searching the cluster is at least as great as searching the periphery: $\underline{c} \geq t$. This is not only compelling, but is required to avoid having a nonsensical equilibrium with a "cluster" consisting of a single firm.[4] Second, the range of valuations relative to periphery search costs is sufficiently great: $h/t \geq 2$. This is necessary to make repeated search in the periphery consistent with consumer optimality. A model in which consumers buy from the first periphery firm sampled does not capture the observation that, in some markets, consumers often sample several stores before buying, nor will such a market support a cluster when $\underline{c} \geq t$. Third, the maximum valuation, $\bar{v}$, is sufficiently great. This ensures that consumers want to engage in search. Finally, the maximum cost of searching the cluster, $\bar{c}$, is sufficiently large. This assumption results in an equilibrium in which some consumers choose to search the periphery exclusively. Since

contains the numbers one through six. All numbers in excess of six are lumped into the category "many" (Flegg, 1983). As another example, the Turkish word for the number "ten" means "many," which suggests that ten was an old limit of counting (Menninger, 1969).

[4] When $\underline{c} < t$, a cluster always exists because all consumers with search costs $c < t$ will go to the cluster. Hence equilibrium may involve a single firm in the "cluster." However, such a case would not shed light on why firms choose to locate near one another.

a maintained hypothesis of our model is that there are many firms in the periphery, it is crucial that equilibrium entail an active periphery. It is worth noting, however, that sufficiently large costs of searching the cluster for some consumers is sufficient but not necessary for having an active periphery. Even if the cost of searching the cluster is small, so that all consumers initially go to the cluster, sufficient product heterogeneity will induce some consumers who are dissatisfied with the offerings at the cluster to continue their search in the periphery. This creates positive demand for periphery firms, which induces some firms to locate there.

3. Price equilibrium

■ In this section we derive optimal consumer search behavior and optimal pricing strategies, holding fixed the number of firms in the periphery and the cluster. We solve for the equilibrium periphery price (Proposition 1), and prove the existence of a unique cluster price equilibrium (Propositions 2 and 3). In Section 4 we characterize the equilibrium number of firms in the cluster and in the periphery.

□ **Periphery.** *Periphery search.* To a consumer in the periphery, the perception of there being an inexhaustible supply of firms to search causes the future to look the same to him regardless of the past: good or bad draws have no bearing on the likelihood of good draws in the future. Thus a consumer for whom search in the periphery is optimal in any period will find periphery search optimal in all subsequent periods.

Consider a consumer who decides to search the periphery. From Kohn and Shavell (1974), repeated search with no recall (which is equivalent to search with recall when previous draws are discarded) and constant search costs from an identically and independently distributed distribution of surpluses yields a reservation utility rule: search until the surplus from purchasing at the current firm is at least as high as the expected gains from search less the search cost. This reservation utility does not depend on the number of searches already made, or on the outcome of those searches. For all consumers, search at a periphery location yields a reservation utility v^R, where v^R is defined by

$$\int_{v^R}^{\bar{v}} [v - v^R] f(v)\, dv = t. \tag{2}$$

Equation (2) says that further periphery search is worthwhile as long as the search cost t does not exceed the expected gains from search, which is the part of the distribution of valuations at least as good as the current draw. v^R is the reservation utility if both the current price and the expected price of the next draw are the same. Consumer c responds to deviations from the anticipated common periphery price of p^P as follows: buy from firm i if and only if $v_i^l \geq v_i^r(p_i)$, where $v_i^r(p_i) = v^R - p^P + p_i$; that is, if the increase (decrease) in p_i over p^P is just offset by an increase (decrease) in the surplus v_i^l.

Solving (2) for the reservation valuation, v^R, we obtain

$$v^R = \bar{v} - \sqrt{2ht}, \tag{3}$$

where, recall, $h \equiv (\bar{v} - \underline{v})$ and measures the degree of heterogeneity in the market. For search to occur at all, v^R must lie in the interval $[\underline{v}, \bar{v}]$. If $v^R \leq \underline{v}$, any draw is acceptable and no search occurs. If $v^R > \bar{v}$, no draw is acceptable and the market will fail to exist. This places restrictions on the value of t relative to h. In particular, the restriction $v^R > \underline{v}$ implies

$$\bar{v} - \sqrt{2ht} > \underline{v}$$

or

$$\bar{v} - \underline{v} = h > \sqrt{2ht} \quad \Rightarrow \quad h/t > 2. \tag{4}$$

Equation (4) will be a standing assumption.

As t decreases, v^R increases: consumers become choosier. In the limiting case, when $t = 0$, $v^R = \bar{v}$, so consumers will continue to shop until they find the variety yielding the highest utility. As $\bar{v}$ approaches $\underline{v}$, h approaches zero and again v^R approaches $\bar{v}$.

Periphery pricing. Since we focus on symmetric equilibria, periphery firm i expects the remaining $(N - 1)$ periphery firms all to charge price p^P. Expected demand for firm i then consists of all consumers who arrive at firm i and find a valuation $v_i^l \geq v_i^r \equiv v^R - p^P + p_i$.

Some consumers who would find firm i's variety acceptable if they sampled firm i will never reach firm i if they find another acceptable variety (that is, a valuation-price combination satisfying the stopping rule) in an earlier search. On any round of search the proportion of consumers finding firm i's product acceptable at price p_i is $[1 - F(v_i^r)]$. The proportion of consumers still searching after j rounds is $F(v^R)^j$, which is the probability of not finding a valuation of at least v^R with price p^P in j consecutive draws from $F(\cdot)$. The demand for firm i's product is then

$$\begin{aligned} D_i^P(p_i; p^P, p^C, n, N) &= \mathcal{L}\Delta\left\{\frac{1}{N}[1 - F(v_i^r)] + \left(\frac{N-1}{N}\right)\left(\frac{1}{N-1}\right)F(v^R)[1 - F(v_i^r)]\right. \\ &\qquad \left. + \left(\frac{N-1}{N}\right)\left(\frac{N-2}{N-1}\right)\left(\frac{1}{N-2}\right)F(v^R)^2[1 - F(v_i^r)] + \dots\right\} \\ &= \frac{\mathcal{L}}{N}\cdot\Delta[1 - F(v_i^r)]\sum_{j=1}^{N} F(v^R)^{j-1}, \end{aligned} \tag{5}$$

where Δ is the proportion of all shoppers who search the periphery, including those who initially searched the cluster but did not buy there.[5] Equation (5) says that demand for periphery firm i is the sum over all N periphery firms of the periphery shoppers who stop at firm i on their first search and buy there, plus those who buy at firm i after one previous search in which no acceptable search was found, and so on. Δ will depend on p^P, p^C, n, and N, but is independent of p_i because consumers cannot observe p_i before deciding to search the periphery. We will solve for Δ later.

Profits to periphery firm i are given by

$$\begin{aligned} \pi_i^P(p_i; p^P, p^C, n, N) &= p_i D_i^P(p_i; p^P, p^C, n, N) \\ &= p_i \frac{\mathcal{L}}{N}\cdot \Delta[1 - F(v_i^r)]\sum_{j=1}^{N} F(v^R)^{j-1}. \end{aligned}$$

The profit-maximizing value of p^P is given by the first-order condition,

[5] Note that while consumers are assumed to perceive the periphery as being limitless, we do allow firms to correctly ascertain the number of periphery firms. This seems plausible, since it is the business of a firm to know how many competitors it has. However, let us note that all qualitative results would be unchanged if we imposed the same perceptions on firms as on consumers.

$$\frac{\mathcal{L}}{N}\cdot \Delta[1 - F(v_i^r)] - \frac{p_i\mathcal{L}\cdot \Delta}{N} f(v_i^r) = 0; \tag{6}$$

since the second-order condition is satisfied,

$$\frac{\partial^2 \pi_i^P}{\partial p_i^2} = -2\frac{\mathcal{L}}{N}\cdot \Delta f(v_i^r) - \frac{p_i\mathcal{L}\cdot \Delta}{N} f'(v_i^r) < 0$$

when $F(\cdot)$ is concave. Recall that we assume F is uniform, which means that $f' = 0$. Hence the profit function of a periphery firm is concave in p_i, so a unique best response exists to each price p^P. By the standard argument (Friedman, 1990) a pure-strategy Nash equilibrium exists in the periphery, given p^C and n.

Proposition 1. $p^P = (2ht)^{1/2}$ and $\partial p^P/\partial h < 0$.

Proof. Solving (6) for p^P when $p_i = p^P$ and hence $v_i^r = v^R = \bar{v} - (2ht)^{1/2}$,

$$\frac{\mathcal{L}\cdot \Delta}{N} = \frac{p^P\mathcal{L}\cdot \Delta}{N[1 - F(v^R)]} f(v^R)$$

or

$$p^P = \frac{1 - F(v^R)}{f(v^R)} = \left[1 - \left(\frac{v^R - \underline{v}}{h}\right)\right]\frac{1}{\frac{1}{h}} \tag{7}$$

$$= \left[\frac{h - (\bar{v} - (2ht)^{1/2} - \underline{v})}{h}\right]\cdot h$$

$$= (2ht)^{1/2}.$$

That $\partial p^P/\partial h > 0$ follows immediately from (7). *Q.E.D.*

The equilibrium periphery price is therefore independent of p^C and n. This is a consequence of the reservation brand property.[6] It is straightforward to calculate that a consumer's expected payoff from searching the periphery is $\bar{v} - (8ht)^{1/2}$ when $p^P = (2ht)^{1/2}$. We assume $\bar{v}$ is sufficiently large so that $\bar{v} - (8ht)^{1/2} > 0$.

□ **Cluster.** *Cluster search.* Define Γ as the proportion of consumers who start search in the cluster. For now Γ is exogenous, but we shall endogenize it later.

A consumer who opts to initially search the cluster will choose to sample all n cluster firms before deciding whether to buy a variety or continue to search. Once the consumer has searched all n firms, he is left with the best draw yielding surplus $u^{\max} = \max_{i\in C}\{v_i^l - p_i\}$. He may buy that variety and obtain $u^{\max}$; opt out of the market for a surplus of zero; or search the periphery. Opting out of the market can never be preferred to periphery search in an equilibrium with a periphery. If periphery search is optimal for any consumer it is preferred to opting out by all consumers (because consumers are *ex ante* identical with respect to the periphery), so the consumer will continue to search by going to the periphery if and only if $u^{\max} < v^R - p^P$ and will buy from the cluster firm yielding $u^{\max}$ otherwise.

Cluster pricing. Unlike the equilibrium periphery price, which is independent of the number of firms in each location, the equilibrium cluster price is a function of the

[6] That p^P depends only on the range of valuations and no other characteristics of $F(\cdot)$, such as its mean and variance, may be peculiar to the uniform distribution.

number of cluster firms.[7] Intuitively, the greater the number of cluster firms, the more intense will be the price competition among them.

In this section we show that a cluster price equilibrium exists for all $h/t \geq 2$ (recall that no search occurs for $h/t < 2$) and that this price is unique. We distinguish between two cases. When $h/t \in [2, 8[n/(n+1)]^2]$, consumers who search the cluster will buy from a cluster firm with certainty. When $h/t > 8[n/(n+1)]^2$, the utility of a cluster shopper who finds his least-preferred variety is sufficiently low that he will prefer to search the periphery rather than buy that product at the cluster price. As a result, not all cluster shoppers end up buying from a cluster firm.

Proposition 2. If $n \geq 2$, $\Gamma > 0$, and $h/t \in [2, 8[n/(n+1)]^2]$ then a unique symmetric cluster equilibrium exists and is defined by $p^C = h/n$.

Proof. Let p^C be a generic symmetric cluster price. The analysis will be partitioned into two cases: (1) $p^C \leq (8ht)^{1/2} - h$; and (2) $p^C > (8ht)^{1/2} - h$. Note that $(8ht)^{1/2} - h > 0$, since $(8ht)^{1/2} - h > 0$ if and only if $h/t < 8$ and it is assumed that $h/t \leq 8[n/(n+1)]^2$.

Case 1. $p^C \leq (8ht)^{1/2} - h$. A consumer's expected surplus from searching the periphery is $\bar{v} - (8ht)^{1/2}$. A consumer who goes to the cluster and buys will receive a surplus of at least $\underline{v} - p^C$. Therefore a consumer who goes to the cluster will buy there for certain if and only if

$$\underline{v} - p^C \geq \bar{v} - (8ht)^{1/2}$$

or

$$(8ht)^{1/2} - h \geq p^C.$$

Under case 1, a firm can expect all consumers who go to the cluster to buy from the cluster firm that offers the highest surplus. If v_i is the valuation a consumer attaches to cluster firm i's product, then this consumer will buy from cluster firm i if and only if

$$v_i - p_i \geq \max\{v_1 - p^C, \ldots, v_{i-1} - p^C, v_{i+1} - p^C, \ldots, v_n - p^C\},$$

where firm i's price is p_i and all other firms charge p^C. Thus the proportion of consumers who go to the cluster and buy from firm i is

$$\int_{\underline{v}}^{\bar{v}} F(v - p_i + p^C)^{n-1} F'(v)\, dv.$$

Given that all other firms are pricing at p^C, firm i's profit from a price of p_i is

$$\pi^C(p_i, p^C, n) = \mathcal{L}\Gamma p_i \int_{\underline{v}}^{\bar{v}} F(v - p_i + p^C)^{n-1} F'(v)\, dv,$$

where Γ is the proportion of all consumers who visit the cluster. Using the assumption that $F(\cdot)$ is a uniform distribution and integrating, one derives

[7] This distinction arises because consumers correctly perceive the number of firms at the cluster but view the periphery as having an unlimited number of firms.

$$\pi^C(p_i, p^C, n) = \begin{cases} \mathcal{L}\Gamma p_i & \text{if } p_i \le p^C - h \\ \mathcal{L}\Gamma p_i \left\{ \left(\dfrac{1}{nh^n} \right) [(h^n - (p^C - p_i)^n] + \left(\dfrac{p^C - p_i}{h} \right) \right\} & \text{if } p_i \in [p^C - h, p^C] \\ [\mathcal{L}\Gamma/(nh^n)] p_i (h - p_i + p^C)^n & \text{if } p_i \in [p^C, p^C + h] \\ 0 & \text{if } p_i \ge p^C + h. \end{cases}$$

Cluster profit is a piecewise function of a firm's own price where the different segments are determined as follows. Since $p_i < p^C - h$ is equivalent to $\bar{v} - p^C < \underline{v} - p_i$, all cluster consumers will buy firm i's product when firm i discounts its price more than h below the price set by the other cluster firms (which is p^C). When instead $p_i > p^C - h$, then the cluster consumers who attach a low valuation to firm i's product but a high valuation to some other cluster firm's product will not buy from firm i. However, as long as $p_i < p^C$ then a consumer will buy for certain from firm i when his valuation of firm i's product is sufficiently great, since $v_i - p_i > \bar{v} - p^C$ for all $v_i \in (\bar{v} - (p^C - p_i), \bar{v}]$. When $p_i > p^C$, even some consumers who highly value firm i's product will not buy it if they assign a sufficiently high valuation to one of the other cluster firms' products, because firm i's product is priced higher. Finally, since $p_i > p^C - h$ is equivalent to $\underline{v} - p^C > \bar{v} - p_i$, firm i's demand is zero when it prices above $p^C + h$.

Though $\pi^C(\cdot)$ is a piecewise function, it is straightforward to show that it is differentiable everywhere. Therefore if p^C is a symmetric cluster equilibrium price, it is defined by $\partial\pi^C(p^C, p^C)/\partial p_i = 0$ or

$$[\mathcal{L}\Gamma/(nh^n)]h^{n-1}[h + p^C - (n + 1)p^C] = 0.$$

Solving yields $p^C = h/n$. To show that h/n is an equilibrium, we must then show that

$$\pi^C\left(\frac{h}{n}, \frac{h}{n}\right) \ge \pi^C\left(p_i, \frac{h}{n}\right) \qquad \forall p_i \ge 0.$$

Since $\partial\pi^C(p_i, h/n)/\partial p_i = \mathcal{L}\Gamma > 0 \ \forall p_i \le (h/n) - h$, we know that the optimal value for p_i lies in $((h/n) - h, (h/n) + h)$. The following lemma establishes that $\pi^C(h/n, h/n) \ge \pi^C(p_i, h/n) \ \forall p_i \in ((h/n) - h, (h/n) + h)$.

Lemma 1. $\partial\pi^C(p_i, p^C)/\partial p_i \gtreqless 0$ as $p_i \lesseqgtr p^C \ \forall p_i \in (p^C - h, p^C + h)$.

Proof. See the Appendix.

To complete case 1 we need to verify that $p^C \le (8ht)^{1/2} - h$ when $p^C = h/n$:

$$\frac{h}{n} \le (8ht)^{1/2} - h$$

or

$$\frac{h}{t} \le 8\left(\frac{n}{n + 1}\right)^2.$$

This holds by assumption. We have then shown that there exists a unique symmetric equilibrium price less than or equal to $(8ht)^{1/2} - h$ and that it equals h/n.

The case of $p^C > (8ht)^{1/2} - h$ is provided in the Appendix. In this case, some consumers who go to the cluster will not buy there but instead prefer to continue search by going to the periphery. It is shown in the Appendix that when $h/t < 8[n/(n + 1)]^2$, there does not exist a symmetric equilibrium in which price exceeds $(8ht)^{1/2} - h$. *Q.E.D.*

When product heterogeneity is not too large relative to the cost of searching the periphery, consumers who visit the cluster will buy there for certain. In that situation, Proposition 2 showed that the equilibrium price equals h/n. The cluster price is then increasing in product heterogeneity and decreasing in the number of cluster firms.

When instead product heterogeneity is sufficiently great relative to periphery search costs, some consumers will end up searching both the cluster and the periphery. For this case, a closed-form solution for the cluster equilibrium price is not available. Proposition 3 does, however, establish the existence and uniqueness of a symmetric equilibrium and provides a characterization of the equilibrium price.

Proposition 3. If $n \geq 2$, $\Gamma > 0$, and $h/t > 8[n/(n + 1)]^2$, then a unique symmetric cluster equilibrium exists and is defined by

$$p^C = \left(\frac{h}{n}\right)\left[1 - \left(\frac{h + p^C - (8ht)^{1/2}}{h}\right)^n\right]. \tag{8}$$

Proof. Let us begin by arguing that if p^C is a symmetric equilibrium price, then $p^C > (8ht)^{1/2} - h$. If not, then $p^C \leq (8ht)^{1/2} - h$, and we know by the proof of Proposition 2 that $p^C = h/n$. However, $h/n \leq (8ht)^{1/2} - h$ if and only if $h/t \leq 8[n/(n + 1)]^2$, which violates an assumption of Proposition 3. Next note that if p^C is a symmetric equilibrium price, then $p^C < (8ht)^{1/2}$. A consumer will never buy at the cluster if $\bar{v} - p^C \leq \bar{v} - (8ht)^{1/2}$. Thus cluster demand and profit are zero if $p^C \geq (8ht)^{1/2}$. This cannot be an equilibrium, as cluster demand and profit are positive for firm i for all $p_i < (8ht)^{1/2}$. We conclude that if p^C is a symmetric equilibrium price, then $(8ht)^{1/2} - h < p^C < (8ht)^{1/2}$.

It is straightforward to show that firm i's profit from pricing at p_i, given that all other cluster firms price at p^C, is

$$\pi^C(p_i, p^C) = \begin{cases} \mathcal{L}\Gamma p_i & \text{if } p_i \leq p^C - h \\ \mathcal{L}\Gamma p_i\left\{\left(\dfrac{1}{nh^n}\right)[h^n - (p^C - p_i)^n] + \left(\dfrac{p^C - p_i}{h}\right)\right\} & \text{if } p_i \in [p^C - h, (8ht)^{1/2} - h] \\ \mathcal{L}\Gamma p_i\left\{\left(\dfrac{1}{nh^n}\right)[h^n - (p^C + h - (8ht)^{1/2})^n] + \left(\dfrac{p^C - p_i}{h}\right)\right\} & \text{if } p_i \in [(8ht)^{1/2} - h, p^C] \\ \mathcal{L}\Gamma p_i\left(\dfrac{1}{nh^n}\right)[(p^C + h - p_i)^n - (p^C + h - (8ht)^{1/2})^n] & \text{if } p_i \in [p^C, (8ht)^{1/2}] \\ 0 & \text{if } p_i \geq (8ht)^{1/2}. \end{cases}$$

If $p_i \leq p^C - h$, then all consumers who go to the cluster buy from firm i. If $p_i > p^C - h$, then only some customers who go to the cluster prefer firm i's product-price combination to that offered by other cluster firms. If $p_i \leq (8ht)^{1/2} - h$, then all consumers who go to the cluster prefer firm i's product-price combination to searching the periphery. Finally, if $p_i > (8ht)^{1/2}$, then all consumers who go to the cluster prefer searching the periphery to firm i's product-price combination. Note that π^C is continuous in p_i.

It is straightforward to derive

$$\frac{\partial \pi^C(p_i, p^C)}{\partial p_i} = \begin{cases} \mathcal{L}\Gamma & \text{if } p_i \leq p^C - h \\ \mathcal{L}\Gamma\left\{\left(\frac{1}{nh^n}\right)[h^n - (p^C - p_i)^n] + \left(\frac{p^C - p_i}{h}\right) + p_i\left(\frac{1}{h^n}\right)(p^C - p_i)^{n-1} - \left(\frac{1}{h}\right)p_i\right\} & \\ \qquad \text{if } p_i \in [p^C - h, (8ht)^{1/2} - h) & \\ \mathcal{L}\Gamma\left\{\left(\frac{1}{nh^n}\right)[h^n - (p^C + h - (8ht)^{1/2})^n] + \left(\frac{p^C - p_i}{h}\right) - \left(\frac{1}{h}\right)p_i\right\} & \\ \qquad \text{if } p_i \in ((8ht)^{1/2} - h, p^C] & \\ \mathcal{L}\Gamma\left\{\left(\frac{1}{nh^n}\right)[(p^C + h - p_i)^n - (p^C + h - (8ht)^{1/2})^n] - p_i\left(\frac{1}{h^n}\right)(p^C + h - p_i)^{n-1}\right\} & \\ \qquad \text{if } p_i \in [p^C, (8ht)^{1/2}] & \\ 0 & \text{if } p_i \geq (8ht)^{1/2}. \end{cases} \tag{9}$$

It is easy to show that $\partial\pi^C/\partial p_i$ exists everywhere except at $p_i = (8ht)^{1/2} - h$. Since then $\partial\pi^C(p^C, p^C)/\partial p_i$ exists, if p^C is a symmetric cluster equilibrium price, it is defined by

$$\partial\pi^C(p^C, p^C)/\partial p_i = 0$$

or

$$h^{n-1}(h - np^C) - [p^C + h - (8ht)^{1/2}]^n = 0. \tag{10}$$

Since we already showed that $p^C > (8ht)^{1/2} - h$, it follows from (10) that $p^C < h/n$.

To prove that (10) defines an equilibrium, we must show that if p^C satisfies (10), then

$$\pi^C(p^C, p^C) \geq \pi^C(p_i, p^C) \qquad \forall p_i \geq 0. \tag{11}$$

Define $\Psi_i(p^C)$ as the optimal price for firm i. Hence we need to show that $\Psi_i(p^C) = p^C$. Since $\partial\pi^C/\partial p_i > 0 \ \forall p_i \leq p^C - h$, then $\Psi_i(p^C) > p^C - h$. Since

$$\pi(p_i, p^C) = 0 \ \forall p_i \geq (8ht)^{1/2}$$

and $\pi^C(p^C, p^C) > 0$, then $\Psi_i(p^C) < (8ht)^{1/2}$. We then need to show that

$$\pi^C(p^C, p^C) \geq \pi^C(p_i, p^C) \qquad \forall p_i \in (p^C - h, (8ht)^{1/2}). \tag{12}$$

Since $\Psi_i(p^C) \in (p^C - h, (8ht)^{1/2})$ and $\partial\pi^C(p^C, p^C)/\partial p_i = 0$, the proof of $\Psi_i(p^C) = p^C$ is completed by showing that firm i's profit function is quasi-concave in its own price for all $p_i \in (p^C - h, (8ht)^{1/2})$.

Lemma 2. If p^C satisfies (10), then $\pi^C(p_i, p^C)$ is quasi-concave in

$$p_i \ \forall p_i \in (p^C - h, (8ht)^{1/2}).$$

Proof. See the Appendix.

To conclude the proof of Proposition 3 we must show that there exists a unique solution to (10).

Lemma 3. There exists a unique value for p^C that satisfies (10).

Proof. See the Appendix.

To summarize the results of this section, we find that there generally exists a unique symmetric equilibrium in prices for the cluster. When the cost of searching the periphery is sufficiently large relative to the degree of product heterogeneity, equilibrium has each firm at the cluster charging a price of h/n. When instead periphery search costs are relatively low, the price charged at the cluster is defined by (8). Note that it is strictly less than h/n. Though there is generally not a closed-form solution for the cluster price in that case, there is such a solution when $n = 2$:

$$p^C = 2[h - (2ht)^{1/2}]\{h^{1/2} - [h - (2ht)^{1/2}]^{1/2}\}.$$

A property that differs between cluster prices and periphery prices is that the former depends on the number of cluster firms while the latter is independent of the number of periphery firms. This distinction is worth discussing. Since search costs once at the cluster are zero, consumers search all cluster firms. This implies that how many firms there are at the cluster affects the likelihood that a consumer who searches a particular cluster firm will buy from it. Thus, the number of cluster firms influences the price that a cluster firm charges. In the periphery, there is a positive cost associated with the search of each firm and, furthermore, consumers perceive there as being an unlimited number of periphery firms. Consumer search behavior is then predicated on their not searching every periphery firm, so that a change in the number of periphery firms does not affect the search rule used by consumers. Since the buying behavior of a consumer who visits a periphery firm is independent of the number of other periphery firms, a periphery firm has no incentive to adjust price depending on how many other periphery firms there are. The key forces at play here are that a consumer searching the cluster anticipates searching all cluster firms before making his choice, while a consumer searching the periphery does not anticipate searching all periphery firms. The latter generates a reservation utility rule that is necessarily independent of the number of periphery firms. While this distinction was derived assuming zero search costs within the cluster and a consumer perception of an unlimited number of periphery firms, it should be robust to allowing for small positive search costs at the cluster and less extreme perceptions about the periphery as long as a consumer finds it very likely that he will search all cluster firms and very unlikely that he will search all periphery firms.

4. Free-entry equilibrium

■ Section 3 described equilibrium in both the cluster and the periphery given the number of firms in each location. We now use this solution to derive the equilibrium number and location of firms. More specifically, we consider an extensive form that is the standard two-stage formulation for modelling entry. In stage 1 there is an unlimited number of potential entrants who simultaneously decide whether to enter the cluster, enter the periphery, or not enter. Given the outcome from stage 1, stage 2 has the entrants simultaneously selecting price and consumers making search decisions. Using the stage 2 equilibrium characterized in the preceding section, we derive a Nash equilibrium for the stage 1 game and thereby characterize a subgame-perfect equilibrium for the two-stage game. We now define a free-entry equilibrium and prove that such an equilibrium exists.

To solve for a firm's profit in the periphery and in the cluster as a function of firms' location decisions, we first need to solve for the search pattern of consumers as a function of the cluster size. That is, we need to determine the proportion of consumers who will search the cluster, which, in terms of our earlier notation, is denoted Γ. Recall that consumers differ in terms of their cost to visiting and searching the cluster. The possible search costs lie in $[\underline{c}, \bar{c}]$ and $G(\cdot)$ is the continuous cumulative density function over $[\underline{c}, \bar{c}]$. Since consumers are otherwise identical, there will exist some consumer type, denoted $\hat{c}$, such that consumer c will initially search the cluster if $c < \hat{c}$ and will skip the cluster and search the periphery if $c > \hat{c}$. In characterizing this marginal consumer $\hat{c}$, we need to consider two cases. Case 1 is when a consumer who visits the cluster buys there for certain and thus optimally chooses not to search the periphery. Case 2 is when some consumers who visit the cluster (and get particularly unattractive draws) choose to continue search by going to the periphery.

Case 1. $h/t \leq 8[n/(n+1)]^2$. In this case, $p^C = h/n$ and all consumers who shop the cluster will buy there. Define $S^C(p^C, n, c)$ as the expected surplus from cluster search. Under case 1,

$$S^C(p^C, n, c) = \int_{\underline{v}}^{\bar{v}} (v - p^C)nF(v)^{n-1}F'(v)\, dv - c$$

$$= \bar{v} - \left(\frac{h}{n+1}\right) - p^C - c.$$

Define S^P as the expected surplus from periphery search. Then

$$S^P = v^R - p^P = \bar{v} - (8ht)^{1/2}.$$

By Propositions 1 and 2, $p^P = (2ht)^{1/2}$ and $p^C = h/n$. Because $\hat{c}$ is defined by the search cost such that $S^C(p^C, n, \hat{c}) = S^P$,

$$\hat{c}(n) = (8ht)^{1/2} - \frac{h}{n+1} - p^C = (8ht)^{1/2} - h\left(\frac{1}{n+1} + \frac{1}{n}\right). \tag{13}$$

Case 2. $h/t > 8[n/(n+1)]^2$. In this case, not all cluster shoppers buy there, so $\max\{\underline{v}, v^R - p^P + p^C\} = v^R - p^P + p^C$ and p^C is defined by Proposition 3. Then

$$S^C(p^C, n, c) = \int_{v^R - p^P + p^C}^{\bar{v}} (v - p^C)nF(v)^{n-1}F'(v)\, dv - c$$

$$= \bar{v} - \left(\frac{h}{n+1}\right) + \left(\frac{h}{n+1}\right)\left[\frac{h - (8ht)^{1/2} + p^C}{h}\right]^{n+1} (h - np^C),$$

while S^P is as before. Then

$$\hat{c}(n) = (8ht)^{1/2} - \left(\frac{h}{n+1}\right) + \left(\frac{h}{n+1}\right)\left[\frac{h - (8ht)^{1/2} + p^C}{h}\right]^{n+1}(h - np^C). \qquad (14)$$

Let us assume that $\hat{c}(n) \in (\underline{c}, \bar{c})$ for all $n \geq 2$, so that we always have an interior solution. For example, this condition is achieved by letting $\underline{c} \cong t$ and having $\bar{c}$ be sufficiently large.

Define $\pi^C(n)$ and $\pi^P(n, N)$ as equilibrium profit for a cluster firm and a periphery firm, respectively, given n firms located at the cluster and N firm located in the periphery. These profit functions take the following form:

$$\pi^C(n) = \mathcal{L}G(\hat{c}(n))[(p^C(n))^2/h] \qquad (15)$$

$$\pi^P(n, N) = \begin{cases} \mathcal{L}[1 - G(\hat{c}(n))]\left[1 - \left(\dfrac{h - (2ht)^{1/2}}{h}\right)^N\right]\dfrac{1}{N}(2ht)^{1/2} \\ \quad \text{if } h/t \in \left[2, 8\left(\dfrac{n}{n+1}\right)^2\right] \\ \mathcal{L}\{[1 - G(\hat{c}(n))] + G(\hat{c}(n))[(p^C(n) + h - (8ht)^{1/2})/h]^n[1 - ((h - (2ht)^{1/2})/h)^N]\}(1/N)(2ht)^{1/2} \\ \quad \text{if } h/t \geq 8\left(\dfrac{n}{n+1}\right)^2, \end{cases} \qquad (16)$$

where $p^C(n)$ is defined in Propositions 2 and 3 and $\hat{c}(n)$ is defined in (13) and (14). Embodied in (16) is that when $h/t \leq 8[n/(n+1)]^2$, the proportion of all consumers who go to the periphery is $[1 - G(\hat{c}(n))]$, as all consumers who initially go to the cluster end up buying at the cluster and thus do not ever search the periphery. When instead $h/t > 8[n/(n+1)]^2$, the proportion of consumers who eventually search the periphery is $[1 - G(\hat{c}(n))] + G(\hat{c}(n))[(h + p^C(n) - (8ht)^{1/2})/h]^n$, where the second term represents those consumers who initially visit the cluster but do not find a satisfactory price-product combination and thus continue search by visiting the periphery. Only a proportion $[1 - (h - (2ht)^{1/2})/h]^N$ of the consumers who go to the periphery end up buying there. Of those consumers, a periphery firm gets a share $1/N$.

Definition. $(n^*, N^*) \in \{0, 2, 3, \ldots\} \times \{0, 1, 2, \ldots\}$ is a free-entry equilibrium if and only if

$$\pi^C(n^*) - K \geq 0 > \pi^C(n^* + 1) - K,$$
$$\pi^P(n^*, N^*) - K \geq 0 > \pi^P(n^*, N^* + 1) - K.$$

These two conditions ensure that each active firm earns at least normal profits and that further entry into the cluster or the periphery is unprofitable. In addition, if these conditions are satisfied, no firm currently in the cluster would want to relocate to the periphery and no firm currently in the periphery would want to relocate to the cluster. If a periphery firm relocated to the cluster, its profit would be $\pi^C(n^* + 1) - K$ (since cluster profits are the same for N^* or $N^* - 1$). However, by the first condition this is nonpositive and thus cannot exceed the firm's profit from remaining at the periphery. Therefore a periphery firm has no reason to move to the cluster. A cluster firm that moves to the periphery would appear to earn $\pi^P(n^* - 1, N^* + 1) - K$. However, this is not stable: the move has created a profitable opportunity for a new firm to enter the cluster and earn $\pi^C(n^*) - K \geq 0$. Presuming such a firm would enter, then the firm that moved from the cluster to the periphery would ultimately earn $\pi^P(n^*, N^* + 1) - K$, which is nonpositive by the second condition. Hence this firm prefers to remain in the cluster.

□ **Existence of a free-entry equilibrium.**

Theorem 1. A free-entry equilibrium exists.

Proof. See the Appendix.

There could exist multiple free-entry equilibria. However, for the case when h/t is sufficiently small and $G(\cdot)$ is weakly concave, it can be shown $\pi^C(n)$ is a single-peaked function of n. In that case, there is a unique equilibrium number of cluster firms.

5. Impact of product heterogeneity

■ Having characterized a free-entry equilibrium, we can now investigate the determinants of the degree of agglomeration. Our focus is on how the parameter h affects agglomeration. Recall that h equals the range of possible valuations that consumers attach to firms' products. Higher values of h correspond to greater product heterogeneity. In addition, h will be larger for markets where the average expenditure is greater. For example, h will be on the order of several thousand dollars for automobiles, a hundred dollars for shoes, and only a few dollars for video rentals.

There are several ways to assess the effect of h on agglomeration. First, one can look at how h affects the likelihood of observing a cluster; that is, the likelihood that there exists an equilibrium with a cluster. Second, given an equilibrium with a cluster, one can look at how h influences the number of cluster firms, n^*. While this measure is useful, it is less than ideal because h also affects the number of periphery firms. If a change in h induces more cluster firms but proportionately more periphery firms, one can reasonably interpret this as a decline in agglomeration. This suggests that a more relevant measure of agglomeration might be the percentage of firms that locate at the cluster, $n^*/(n^* + N^*)$. Yet another measure is the proportion of shoppers who visit the cluster, $G(\hat{c}(n^*))$. All of these measures are considered in our analysis, though some are considered only in the simulations.

To begin, let us consider the effect of product heterogeneity on cluster profit when h/t is moderate, holding the number of cluster firms fixed. First note that the cluster price, h/n, is rising in h. This effect clearly raises the profit from joining the cluster. Next note that greater product heterogeneity results in consumers engaging in more search. *Ceteris paribus,* this induces more consumers to visit the cluster and thereby raises cluster demand. However, a rise in h also induces cluster firms to price higher. Since consumers anticipate this higher price, this will discourage them from going to the cluster. The net effect of h on cluster demand is then unclear. Further complicating matters, the periphery price, $(2ht)^{1/2}$, is also rising in h as periphery firms take further advantage of their local monopoly power. Of particular significance, the price of a cluster firm relative to a periphery firm, $(1/n)(h/2t)^{1/2}$, is rising in h. Hence the price at the cluster is rising faster than the price in the periphery. This rise in relative prices counteracts the increased incentive to search and leaves in doubt whether cluster demand will rise in response to a rise in h. The *a priori* effect of h on cluster demand and thus cluster profit is then unclear.

Theorem 2 shows that when h/t is moderate, cluster profit is strictly increasing in h because the number of consumers who visit the cluster is increasing in h. For this part of the parameter space, the net effect of a rise in product heterogeneity is to raise the profit from joining the cluster.

Theorem 2. There exists $\theta > 2$ such that if $h/t \in [2, \theta]$, then $\partial\pi^C(n)\partial h > 0$ for all $n \geq 2$.

Proof. Restrict $h/t \leq 8[n/(n + 1)]^2\ \forall n$. This implies that

$$\pi^C(n) = \pounds G(\hat{c})\left(\frac{h}{n^2}\right) \qquad \forall n \geq 2,$$

where, from (13),

$$\hat{c} = (8ht)^{1/2} - h\left[\left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right)\right]$$

$$\frac{\partial\pi^C}{\partial h} = \pounds G'(\hat{c})\left[\left(\frac{2t}{h}\right)^{1/2} - \left(\frac{1}{n+1}\right) - \left(\frac{1}{n}\right)\right] + \pounds G(\hat{c})\left(\frac{1}{n^2}\right).$$

Thus $\partial\pi^C/\partial h > 0$ if

$$\begin{gathered}
\left(\frac{2t}{h}\right)^{1/2} \geq \left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right) \\
\frac{1}{\left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right)} \geq \left(\frac{h}{2t}\right)^{1/2} \\
\left[\frac{1}{\left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right)}\right]^2 \geq \frac{h}{2t} \\
2\left[\frac{1}{\left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right)}\right]^2 \geq \frac{h}{t}.
\end{gathered} \tag{17}$$

Since

$$2\left[\frac{1}{\left(\frac{1}{n+1}\right) + \left(\frac{1}{n}\right)}\right]^2 > 2 \qquad \forall n \geq 2,$$

then there exists $\theta > 2$ such that (17) holds $\forall n\ \forall(h/t) \in [2, \theta]$. Hence

$$\partial\pi^C/\partial h > 0\ \forall(h/t) \in [2, \theta].$$

Q.E.D.

Since an equilibrium with a cluster exists if and only if there exists $n^0 \geq 2$ such that $\pi^C(n^0) - K \geq 0$, it follows from Theorem 2 that greater heterogeneity makes it more likely for a cluster to be present. Formally, let us define $\underline{\mathcal{L}}$ as the minimum size of the market such that a cluster equilibrium exists:

$$\underline{\mathcal{L}} \equiv \min\{\mathcal{L} \in \Re_+ \,|\, \exists n^0 \geq 2 \text{ subject to } \pi^C(n^0) - K \geq 0\}.$$

Corollary 1. $\partial\underline{\mathcal{L}}(h)/\partial h < 0$. As h increases, it takes a smaller market to support a cluster. Furthermore, it follows from Theorem 2 that the size of the cluster is nondecreasing in heterogeneity. Because of the possibility that there are multiple equilibrium cluster sizes, we shall state the result in terms of $\bar{n}^*$, the largest equilibrium cluster size: $\bar{n}^* \equiv \max\{n \in \{2, 3, \ldots\} \,|\, \pi^C(n) - K \geq 0 \geq \pi^C(n+1) - K\}$.

Corollary 2. $\partial\bar{n}^*/\partial h \geq 0$. When $h/t \leq \theta$, $\partial\hat{c}/\partial h > 0$, so that increasing h raises the cluster price *and* brings more consumers to the cluster. Hence cluster profit rises. More generally, it is possible for $\partial\hat{c}/\partial h < 0$. It depends on how fast the cluster price is rising vis-à-vis the periphery price. If the former rises much faster, then few consumers may go to the cluster when heterogeneity is greater. In that case, whether cluster profit rises depends on the relative size of $\partial\hat{c}/\partial h$ and $\partial p^C/\partial h$. To examine this more generally, we will use simulations.

In terms of three of our measures of agglomeration—the likelihood of a cluster being viable, the number of cluster firms, and the proportion of shoppers who go to the cluster—we find that greater product heterogeneity results in greater agglomeration. The other proposed measure was the proportion of all firms that locate at the cluster. For this measure, it is insufficient to show that cluster profit is rising in h. One must instead show that cluster profit is rising faster than periphery profit. We have not found it to be analytically tractable to evaluate how h affects relative profits. To handle this measure and to handle general values for h/t, we also turn to simulations.

□ **Numerical simulations.** We solve the model numerically for various parameter values to explore the effect of increasing heterogeneity on the proportion of firms locating in the cluster (that is, $n^*/(N^* + n^*)$) and the proportion of cluster shoppers (that is, $G(\hat{c}(n^*))$). For each set of simulations we set $\underline{c} = t = 1$ and increase h from 2 to 12. Thus $h/t \geq 2$ in each case. Note that as we let h increase from 2 to 12 we shall move from equilibria in which no consumers visit both the cluster and the periphery, which occurs when $h/t < 8[n/(n+1)]^2$, to equilibria in which some consumers search both the cluster and the periphery, which occurs when $h/t > 8[n/(n+1)]^2$. Since $8[n/(n+1)]^2 < 8 \; \forall n$, values of $h \geq 8$ always ensure that $h/t > 8[n/(n+1)]^2$.

Table 2 shows $n^*/(N^* + n^*)$ for $G(c)$ uniform and $\bar{c} \in \{10, 12.5, 15, 17.5, 20\}$.[8] Table 3 uses a concave[9] function $G(c)$ of the form $G(c) = 1 - e^{-\gamma(\hat{c}-\underline{c})}$, where γ is a measure of the concavity of $G(\cdot)$. We solve for the equilibrium industry structure for each value of γ in $\{.05, .10, .15, .20, .25\}$. As Tables 2 and 3 show, the proportion of cluster firms is generally rising in h but is not strictly monotonic. Since both N^* and n^* grow with h, the numerator of $n^*/(N^* + n^*)$ increases with h, but the denominator can increase more rapidly. Table 2 shows a nonincreasing relationship between $n^*/(N^* + n^*)$ and $\bar{c}$, holding h fixed, confirming the intuition that cluster shopping, and hence locating in the cluster, is less attractive as consumers, on average, find cluster shopping more expensive. Table 3 shows that the proportion of cluster firms rises for each value of h

[8] These values were chosen to illustrate how $n^*/(N^* + n^*)$ changes with $\bar{c}$. For $\bar{c}$ substantially below 10, cluster demand is too great to support a large periphery, so our model is not applicable. For $\bar{c}$ substantially above 20, the proportion of cluster firms falls to zero for all h, and again this model is not applicable.

[9] We have substantially similar results for when $G(\cdot)$ is a strictly convex function: $(\hat{c} - \underline{c})^2/(\bar{c} - \underline{c})^2$. Since the properties of the equilibrium number of firms do not change qualitatively from the uniform case, we do not report these results here.

TABLE 2 **Proportion of Firms in Cluster (Uniform G)**

h	$\bar{c} = 10$	$\bar{c} = 12.5$	$\bar{c} = 15$	$\bar{c} = 17.5$	$\bar{c} = 20$
2	.0000	.0000	.0000	.0000	.0000
3	.0909	.0870	.0000	.0000	.0000
4	.1304	.0741	.0000	.0000	.0000
5	.1667	.1071	.0667	.0000	.0000
6	.1538	.1429	.0968	.0000	.0000
7	.1923	.1333	.1250	.0882	.0000
8	.2400	.1667	.1176	.1143	.0789
9	.2800	.1935	.1471	.1081	.1053
10	.2800	.1935	.1389	.1316	.1000
11	.3333	.2258	.1667	.1282	.0952
12	.3913	.2188	.1622	.1538	.0976

as the cumulative density function on cluster search costs puts more mass on lower values for c. Reading down Table 3, we find that the proportion of cluster firms generally, but not always, increases with h. While the relationship between $n^*/(N^* + n^*)$ and h is not strictly monotonic, this is because the integer constraint on the number of firms and the larger number of firms in the periphery occasionally keep n^* fixed for an increase in h that increases N^*.

Table 4 presents the proportion of all shoppers who visit the cluster for a uniform $G(\cdot)$ for the same parameters as in Table 2: h takes on integer values from 2 to 12 and $\bar{c}$ ranges from 10 to 20. Increasing $\bar{c}$ for a fixed h lowers $G(\hat{c}(n^*))$, while increasing h generally increases $G(\hat{c}(n^*))$, indicating that cluster shopping is generally more active when there is greater product heterogeneity.

Table 5 shows how p^C varies with h and $\bar{c}$ for a uniform $G(\cdot)$. The equilibrium cluster price is monotonically nondecreasing in $\bar{c}$. However, there is no apparent relationship between $p^C(n^*)$ and h. As h increases, p^C decreases holding n^* constant, but

TABLE 3 **Proportion of Firms in Cluster (Concave G)**

h	$\gamma = .05$	$\gamma = .10$	$\gamma = .15$	$\gamma = .20$	$\gamma = .25$
2	.0000	.0000	.0000	.1176	.1250
3	.0000	.0000	.0952	.2000	.1765
4	.0000	.0769	.1364	.2105	.2353
5	.0000	.1111	.1739	.2500	.2778
6	.0000	.1379	.2083	.2381	.3333
7	.0000	.1333	.2000	.2857	.3333
8	.0000	.1613	.2308	.3182	.3889
9	.0750	.1563	.2308	.3182	.4211
10	.0698	.1818	.2593	.3636	.4211
11	.0930	.1765	.2593	.3636	.4737
12	.0889	.2000	.2857	.4091	.4737

TABLE 4 **Proportion of Cluster Shoppers (Uniform G)**

h	$\bar{c} = 10$	$\bar{c} = 12.5$	$\bar{c} = 15$	$\bar{c} = 17.5$	$\bar{c} = 20$
2	.000	.000	.000	.000	.000
3	.155	.122	.000	.000	.000
4	.258	.116	.000	.000	.000
5	.342	.209	.091	.000	.000
6	.359	.281	.174	.000	.000
7	.435	.290	.238	.202	.000
8	.503	.354	.243	.207	.128
9	.564	.409	.299	.209	.182
10	.585	.422	.306	.260	.183
11	.643	.473	.356	.264	.184
12	.696	.486	.363	.308	.233

$\partial\bar{n}^*/\partial h \geq 0$ (Corollary 2) and $\partial p^C/\partial n^* < 0$. Thus, taking into account changes in the equilibrium number of cluster firms, dp^C/dh is ambiguous.

The conclusions we draw from our analytical results and numerical analysis are as follows. There are counteracting forces at play when it comes to assessing how product heterogeneity influences the extent to which firms agglomerate. Greater product heterogeneity makes search more worthwhile which, *ceteris paribus,* will increase the number of consumers who visit the cluster. This rise in cluster demand and fall in periphery demand should result in more firms locating at the cluster and fewer firms locating in the periphery. However, changing product heterogeneity also affects the prices that firms will charge. The periphery price is increasing in product heterogeneity as periphery firms take greater advantage of their local monopoly power. For moderate values of h/t, the cluster price is also increasing in h. For more general values of h, we were unable to sign the effect of h on p^C, though simulations (not shown here) always show it to be increasing in h. How the effect of h on prices affects the profit

TABLE 5 **Cluster Price (Uniform G)**

h	$\bar{c} = 10$	$\bar{c} = 12.5$	$\bar{c} = 15$	$\bar{c} = 17.5$	$\bar{c} = 20$
2	—	—	—	—	—
3	1.500	1.500	—	—	—
4	1.333	1.986	—	—	—
5	1.250	1.666	2.387	—	—
6	1.500	1.500	1.989	—	—
7	1.400	1.748	1.748	2.293	—
8	1.333	1.599	1.992	1.992	2.577
9	1.286	1.500	1.798	2.231	2.231
10	1.428	1.666	1.995	1.995	2.462
11	1.375	1.571	1.832	2.189	2.686
12	1.333	1.714	1.996	1.996	2.381

of being a cluster firm relative to the profit of being a periphery firm is ambiguous. For moderate values of h/t, the cluster price rises faster than the periphery price, which increases the relative attractiveness of agglomeration. However, if the cluster price rises too fast, this can curtail the number of consumers who visit the cluster and thus cluster demand. This will tend to reduce the relative profitability of joining the cluster.

In spite of these various forces, there is a strong tendency within the model for there to be more clustering in markets with more heterogeneous products. This is reflected in the result that the minimum market size required for a cluster to be present is smaller the higher is h. The number of cluster firms is increasing in h. And, finally, the relative proportion of cluster firms among all firms in the market is generally increasing in h, though the relationship is not strictly monotonic.

In concluding this section, let us discuss the implications of our assumption that the cost of searching the periphery and the price charged by periphery firms are independent of the number of periphery firms. This assumption was made to improve the tractability of the model and would seem to be a reasonable approximation if one is considering markets with a large number of periphery firms so that the addition or subtraction of a few firms is likely to have a small impact. Nevertheless, in actuality, increasing the number of periphery firms would reduce the average distance to a periphery firm and thereby both lower the cost of searching the periphery and intensify price competition among periphery firms. How would our main result—the proportion of firms at the cluster is increasing in the degree of product heterogeneity—change if these effects were allowed for? We know from our earlier analysis that a rise in product heterogeneity causes the number of periphery firms to increase. If we now allow search costs to adjust downward, this will increase periphery demand but also lower prices in the periphery. The former effect increases the profits of a periphery firm, while the latter reduces them. If the latter effect dominates, then the periphery does not expand as much, which would reinforce our conclusion; if the former effect dominates, then our conclusion comes into question. *A priori* it is unclear which effect will dominate. However, it is our maintained hypothesis that for the markets we have in mind, these effects are likely to be second order, relative to the effects taking place at the cluster. We imagine there being many more periphery firms than firms at the cluster, so that a few more firms in the periphery is expected to have a much smaller effect than another firm at the cluster. Whether this conjecture is borne out must await further research.

6. Concluding remarks

■ In this article we attempt to explain the tendency for firms selling similar products to locate near one another. The key concepts underlying the model are that (1) products exhibit sufficient heterogeneity that consumers desire to search multiple firms before buying, and so nearby firms do not compete too intensively in price; (2) consumer search costs are positive, so consumers typically search fewer than the total number of available stores; and (3) there exist many potential entrants to the market. The presence of many potential entrants limits the number of firms that will locate in the cluster, generating a "periphery" of unclustered firms. Positive search costs give rise to strategic search decisions on the part of consumers; this, in turn, makes the choice of firm location important.

While the model is necessarily stylized, we believe it captures the main observations as summarized in Table 1: markets with products exhibiting greater product heterogeneity and/or higher average prices tend to be more clustered, but some firms always remain outside the cluster(s). The model predicts that, given the number of firms in each location, prices in both the periphery and the cluster increase with the degree of heterogeneity, reflecting the lower degree of substitutability across varieties.

In addition, cluster prices decrease with the number of cluster firms, reflecting the greater number of choices for cluster shoppers and hence the greater likelihood that a rival's variety will satisfy a shopper. For moderate amounts of product heterogeneity, greater heterogeneity also increases the likelihood a cluster will exist; if one exists, the cluster grows with h. The numerical simulations demonstrated that increases in heterogeneity tend to increase cluster demand and hence the proportion of firms locating in the cluster. The need to resort to simulations reflects the complexity of the model: both location and price are choice variables for firms, while consumers choose search patterns optimally. This complexity, we believe, is an important innovation in the present model over previous work in this area.

Previous work, in particular Eaton and Lipsey (1979), Stahl (1982), Wolinsky (1983), and Dudey (1990), has emphasized the conditions under which a single cluster exists or, in the case of Dudey, under which multiple clusters of the same size exist. Our analysis, supporting the empirical observations of Table 1, suggests that unclustered firms will remain in equilibrium for some ranges of parameter values. These periphery firms are profitable both when the ratio of heterogeneity to cluster search costs is small—consumers prefer to forgo the benefits of multiple searches at the cluster in order to save the cluster search cost—and when the ratio is large. At first this may appear counterintuitive. A high degree of heterogeneity, relative to cluster search costs, increases the attractiveness of the cluster to consumers. With sufficient demand at the cluster, periphery firms would appear to be unprofitable. However, periphery firms sell to consumers who fail to find an acceptable product at the cluster, so demand is always positive, and periphery firms do not face the same degree of price competition as do cluster firms.

One aspect of shopping behavior that the model does not address is that of multipurpose shopping trips. Consumers often buy more than one product at a time, so firms selling dissimilar products may choose to locate near one another, increasing demand by allowing consumers to save on their overall costs of shopping. Consumers may learn about the prices charged by cluster firms merely through repeated trips to the cluster. Casual observation suggests that this is an important component of mall shopping.

Firms typically sell more than one brand of an item, which this model does not allow. Selling multiple brands may help firms forestall search, raising the price they can profitably charge. In addition, periphery firms offering several brands may become more attractive, relative to the cluster, by lowering the expected cost of search. A profitable strategy for a periphery firm may be to take advantage of lower land prices by building a large store and stocking many items, à la Wal-Mart. Cluster firms, facing high mall prices and low intracluster search costs, may instead focus on a smaller range of product offerings.

Despite these concessions for the sake of tractability, our model highlights the importance of product heterogeneity in both search and location decisions.

Appendix

■ Proofs of Lemmas 1, 2, and 3, Theorem 1, and the remainder of Proposition 2 follow.

Proof of Lemma 1. First note that we can represent cluster profit as $\pi^C(p_i, p^C) = p_i D_i(p_i, p^C)$, where

$$D_i(p_i, p^C) = \mathcal{L}\Gamma\left\{\left(\frac{1}{nh^n}\right)[h^n - (p^C - p_i)^n] + \frac{p^C - p_i}{h}\right\} \qquad \forall p_i \in (p^C - h, p^C).$$

Then

$$\frac{\partial D_i}{\partial p_i} = -\mathcal{L}\Gamma\left\{\frac{1}{h} - \left(\frac{1}{h^n}\right)(p^C - p_i)^{n-1}\right\} < 0 \qquad \forall p_i \in (p^C - h, p^C]$$

$$\frac{\partial^2 D_i}{\partial p_i^2} = -\mathcal{L}\Gamma\left(\frac{n-1}{h^n}\right)(p^C - p_i)^{n-2} < 0 \qquad \forall p_i \leq p^C.$$

Therefore,

$$\frac{\partial^2 \pi^C}{\partial p_i^2} = 2\left(\frac{\partial D_i}{\partial p_i}\right) + \frac{\partial^2 D_i}{\partial p_i^2} < 0 \qquad \forall p_i \in (p^C - h, p^C].$$

Since $\partial\pi^C(p^C, p^C)/\partial p_i = 0$ then $\partial\pi^C(p_i, p^C)/\partial p_i > 0$ $\forall p_i \in (p^C - h, p^C)$.

For $p_i \in (p^C, p^C + h)$,

$$\frac{\partial \pi^C}{\partial p_i} = [\mathcal{L}\Gamma/(nh^n)](h - p_i + p^C)^{n-1}[h + p^C - (n+1)p_i].$$

Hence $\partial\pi^C/\partial p_i < 0$ if and only if $(n+1)p_i > h + p^C$. Therefore Lemma 1 is true if and only if $(n+1)p^C \geq h + p^C$ or $p^C \geq h/n$, which is indeed true since $p^C = h/n$. *Q.E.D.*

Remainder of proof of Proposition 2. Case 2. $p^C > (8ht)^{1/2} - h$. If $p^C > (8ht)^{1/2} - h$, then consumers who receive sufficiently low valuations at the cluster prefer not to buy and instead search the periphery. In particular, a consumer who evaluates firm i's product at v_i will prefer to buy it rather than search the periphery if and only if $v_i - p_i \geq \bar{v} - (8ht)^{1/2}$. Therefore firm i's profit is

$$\mathcal{L}\Gamma p_i \int_{s(p_i)}^{\bar{v}} F(v - p_i + p^C)^{n-1}F'(v)\,dv, \qquad s(p_i) \equiv \max\{\bar{v} - (8ht)^{1/2} + p_i, \underline{v}\}.$$

The expression for a cluster firm's profit, $\pi^C(p_i, p^C)$, is provided in the proof of Proposition 3. There it is shown that π^C is differentiable at $p_i = p^C$. Hence if p^C is a symmetric equilibrium price, it is defined by the first-order condition $\partial\pi^C(p^C, p^C)/\partial p_i = 0$. Under the supposition that $p^C > (8ht)^{1/2} - h$, $s(p^C) = \bar{v} - (8ht)^{1/2} + p^C$. Therefore the first-order condition takes the form

$$\frac{\partial}{\partial p_i}\left\{\mathcal{L}\Gamma p_i \int_{\bar{v}-(8ht)^{1/2}+p_i}^{\bar{v}} \left[\frac{v - p_i + p^C - \underline{v}}{h}\right]^{n-1}\left(\frac{1}{h}\right)dv\right\}\Bigg|_{p_i=p^C} = 0.$$

Integrating and taking the derivative with respect to p_i, one derives that p^C is defined by

$$h^{n-1}(h - np^C) - [p^C - ((8ht)^{1/2} - h)]^n = 0.$$

Since, by presumption, $p^C > (8ht)^{1/2} - h$, then it follows that $h - np^C > 0$ or $p^C < h/n$. We can now establish a contradiction. Since $h/n > p^C$ and $p^C > (8ht)^{1/2} - h$, then $h/n > (8ht)^{1/2} - h$. It follows that $h/t > 8[n/(n+1)]^2$, which contradicts our assumption. We conclude that there does not exist a symmetric equilibrium with $p^C > (8ht)^{1/2} - h$ when $(h/t) \leq 8[n/(n+1)]^2$. *Q.E.D.*

Proof of Lemma 2. Note that the expressions for π^C when $p_i \in [p^C - h, p^C]$ and $h/t \leq 8[n/(n+1)]^2$ (see the proof of Proposition 2) and when $p_i \in (p^C - h, (8ht)^{1/2} - h]$ and $h/t \geq 8[n/(n+1)]^2$ are identical. Lemma 1 showed that this expression is increasing in p_i $\forall p_i \in (p^C - h, p^C)$. It follows that

$$\partial\pi^C/\partial p_i > 0 \ \forall p_i \in (p^C - h, (8ht)^{1/2} - h).$$

To complete the proof of Lemma 2, we shall show that

$$\frac{\partial \pi^C(p_i, p^C)}{\partial p_i} \gtreqless 0 \qquad \text{as} \qquad p_i \lesseqgtr p^C \qquad \forall p_i \in ((8ht)^{1/2} - h, (8ht)^{1/2}).$$

Since $(\partial^2\pi^C/\partial p_i^2) = -\mathcal{L}\Gamma(2/h)$ $\forall p_i \in ((8ht)^{1/2} - h, p^C)$ and $\partial\pi^C(p^C, p^C)/\partial p_i = 0$, then

$$\partial\pi^C(p_i, p^C)/\partial p_i > 0 \qquad \forall p_i \in ((8ht)^{1/2} - h, p^C).$$

Next note that for $p_i \in (p^C, (8ht)^{1/2})$,

$$\frac{\partial\pi^C}{\partial p_i} < 0 \text{ if and only if } [p^C + h - (8ht)^{1/2}]^n > (p^C + h - p_i)^{n-1}[p^C + h - (n + 1)p_i].$$

Hence if $p_i \geq (p^C + h)/(n + 1)$, then $\partial\pi^C/\partial p_i < 0$. Next note that for $p_i \in (p^C, (8ht)^{1/2})$,

$$\frac{\partial^2\pi^C}{\partial p_i^2} = -\mathcal{L}\Gamma\left(\frac{1}{h^n}\right)(p^C + h - p_i)^{n-2}[2(p^C + h) - (n + 1)p_i]. \tag{A1}$$

Hence $\partial^2\pi^C/\partial p_i^2 < 0$ if and only if $p_i \leq 2(p^C + h)/(n + 1)$. Since $\partial\pi^C(p^C, p^C)/\partial p_i = 0$, then

$$\partial\pi^C/\partial p_i < 0 \ \forall p_i \in (p^C, 2(p^C + h)/(n + 1)].$$

We showed from (A1) that $\partial\pi^C/\partial p_i < 0$ when $p_i \geq (p^C + h)/(n + 1)$. Thus

$$\partial\pi^C/\partial p_i < 0 \qquad \forall p_i \in [(p^C + h)/(n + 1), (8ht)^{1/2}].$$

Combining these results, we conclude that $\partial\pi^C/\partial p_i < 0 \ \forall p_i \in (p^C, (8ht)^{1/2})$. *Q.E.D.*

Proof of Lemma 3. For this purpose, define

$$\phi(p^C) \equiv h^{n-1}(h - np^C) - [p^C + h - (8ht)^{1/2}]^n.$$

First note that

$$\phi((8ht)^{1/2} - h) = h^{n-1}[h - n((8ht)^{1/2} - h)].$$

$\phi((8ht)^{1/2} - h) > 0$ if and only if

$$h > n[(8ht)^{1/2} - h]$$

or

$$\frac{h}{t} > 8\left(\frac{n}{n + 1}\right)^2,$$

which holds by assumption. Next note that

$$\phi(h/n) = -[(h/n) - ((8ht)^{1/2} - h)] < 0.$$

Since $\phi(\cdot)$ is continuous, it follows from $\phi((8ht)^{1/2} - h) > 0 > \phi(h/n)$ that there exists $p^C \in ((8ht)^{1/2} - h, h/n)$ such that $\phi(p^C) = 0$. Finally, p^C is unique as

$$\phi'(p^C) = -h^{n-1}n - n[p^C + h - (8ht)^{1/2}]^{n-1} < 0.$$

Q.E.D.

Proof of Theorem 1. To begin, let us establish two properties of $\pi^C(\cdot)$ and $\pi^P(\cdot)$. Recall that

$$\pi^C(n) = \mathcal{L}G(\hat{c}(n))\left[\frac{(p^C(n))^2}{h}\right].$$

Since $G(\hat{c}(n)) \in [0, 1] \ \forall n$, then

$$\lim_{n\to\infty} \pi^C(n) = 0 \qquad \text{if and only if } \lim_{n\to\infty} p^C(n) = 0.$$

If $h/t \leq 8[n/(n + 1)]^2$, then $p^C(n) = h/n$. Hence if $h/t < 8$, then

$$\lim_{n\to\infty} p^C(n) = \lim_{n\to\infty}\frac{h}{n} = 0.$$

If $h/t > 8[n/(n + 1)]^2$, then p^C is defined by

$$p^C(n) = \left(\frac{h}{n}\right)\left[1 - \left(\frac{p^C(n) + h - (8ht)^{1/2}}{h}\right)^n\right].$$

Since $p^C(n) < h/n$, it follows that

$$\lim_{n\to\infty} p^C(n) \leq \lim_{n\to\infty}\frac{h}{n} = 0.$$

We have then shown that $\lim_{n\to\infty} \pi^C(n) = 0$.

Next let us show that $\lim_{N\to\infty} \pi^P(n, N) = 0\ \forall n$.

$$\pi^P(n, N) = \mathcal{L}\Delta(n, N)(2ht)^{1/2}\left(\frac{1}{N}\right),$$

where $\Delta(n, N) \in [0, 1]$ is the equilibrium proportion of consumers who buy from a representative periphery firm.

$$\lim_{N\to\infty} \pi^P(n, N) = \lim_{N\to\infty} \mathcal{L}\Delta(n, N)(2ht)^{1/2}\left(\frac{1}{N}\right) = 0.$$

Case 1. $\pi^C(n) - K \leq 0\ \forall n \geq 2$. First suppose that $\pi^P(0, N) - K \leq 0\ \forall N \geq 1$. Then $(n, N) = (0, 0)$ is a free-entry equilibrium.

Now suppose that there exists $N^0 \geq 1$ such that $\pi^P(0, N^0) - K > 0$. Since

$$\lim_{N\to\infty} \pi^P(0, N) - K = -K < 0,$$

there exists $N^* \geq N^0$ such that

$$\pi^P(0, N^*) - K \geq 0 \geq \pi^P(0, N^* + 1) - K.$$

Then $(n, N) = (0, N^*)$ is a free-entry equilibrium.

Case 2. There exists $n^0 \geq 2$ such that $\pi^C(n^0) - K > 0$. Since $\lim_{n\to\infty} \pi^C(n) - K = -K < 0$, then there exists $n^* \in \{n^0, \ldots\}$ such that $\pi^C(n^*) - K \geq 0 \geq \pi^C(n^* + 1) - K$. If $\pi^P(n^*, N) - K \leq 0\ \forall N \geq 1$, then $(n, N) = (n^*, 0)$ is a free-entry equilibrium. If instead there exists $N^0 \geq 1$ such that $\pi^P(n^*, N^0) - K > 0$, there exists $N^* \in \{N^0, N^0 + 1, \ldots\}$ such that $\pi^P(n^*, N^*) - K \geq 0 \geq \pi^P(n^*, N^* + 1) - K$. Hence (n^*, N^*) is a free-entry equilibrium. *Q.E.D.*

References

DUDEY, M. "Competition by Choice: The Effect of Consumer Search on Firm Location Decisions." *American Economic Review,* Vol. 80 (1990), pp. 1092–1105.

EATON, B.C. AND LIPSEY, R.G. "Comparison Shopping and the Clustering of Homogenous Firms." *Journal of Regional Science,* Vol. 19 (1979), pp. 421–435.

FISCHER, J.H. "Consumer Search and Firm Clustering with Heterogeneous Goods." Ph.D. dissertation, Department of Economics, Johns Hopkins University, 1992.

FLEGG, G. *Numbers: Their History and Meaning.* New York: Schocken Books, 1983.

FRIEDMAN, J.W. *Game Theory with Applications to Economics.* 2d ed. New York: Oxford University Press, 1990.

KOHN, M.G. AND SHAVELL, S. "The Theory of Search." *Journal of Economic Theory,* Vol. 9 (1974), pp. 93–123.

MENNINGER, K. *Number Words and Number Symbols: A Cultural History of Numbers.* Cambridge, Mass.: MIT Press, 1969.

STAHL, K. "Differentiated Products, Consumer Search, and Locational Oligopoly." *Journal of Industrial Economics,* Vol. 31 (1982), pp. 97–113.

WOLINSKY, A. "Retail Trade Concentration Due to Consumers' Imperfect Information." *Bell Journal of Economics,* Vol. 14 (1983), pp. 275–282.