

# Urban Agglomeration and Dispersion: A Synthesis of Alonso and Krugman\*

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Urban agglomeration becomes increasingly important because of the globalization of world economies. This paper is a general equilibrium analysis of urban agglomeration economies due to product variety, and agglomeration diseconomies due to intra-city congestion in a two-city system framework. Special attention is paid to the impacts of transportation cost decrease on urban concentration and dispersion. Our main result is that dispersion necessarily takes place when the transportation cost is sufficiently low. We also conduct numerical calculations using specific parameter values, and depict a structural transition from dispersion to agglomeration, and then re-dispersion when the transportation costs decrease monotonically over time. Finally, we observe that dispersion is usually bad as compared to agglomeration, from a welfare point of view. © 1998 Academic Press

*Key Words:* Population dispersion, product variety, urban agglomeration

## 1. INTRODUCTION

Standard textbooks on urban economics such as Mills and Hamilton [8] state that the existence of urban areas is explained in terms of increasing returns to scale at the city level, which is referred to as urban agglomeration economies comprising localization economies and urbanization economies. Agglomerating in a city, firms can exchange information by face-to-face communications and reduce various kinds of transaction costs between firms. In addition, consumers can enjoy easy access to a variety of differentiated products. These are typical positive externalities inducing urban concentration of firms and consumers.

On the other hand, there are counter-forces to agglomeration in urban activities. Excess concentration in large cities brings negative externalities

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due to congestion, such as longer commuting costs and scarce land for housing and offices. These space constraints work as a dispersion force. A level of urban concentration is therefore determined by a balance between the agglomeration force and the dispersion force.

Several factors exert an influence on the relative strength between the agglomeration and dispersion forces. Among them, we consider the most important one to be the interregional transportation costs of outputs, which have been decreasing over time relative to other costs and prices due to technological progress and improvements in transportation facilities. It will be revealed later that a decrease in transportation costs results either in agglomeration or dispersion.

Recently, Krugman [6] developed a two-region general equilibrium model in a monopolistic competition framework. It enabled us to analyze the impacts of a change in the interregional transportation costs on the degree of urban agglomeration. He demonstrated that with high transportation costs firms and workers disperse because the dispersion force is dominant relative to the agglomeration force, whereas they agglomerate with low transportation costs. This implies that agglomeration tends to prevail, since we expect reduction in transportation costs due to technical progress.

In fact, while rural areas lost population, large cities emerged all over the world after the Industrial Revolution. However, such concentration ceased or dispersion has been taking place after 1970 in most developed countries, as documented by Vining et al. [16].

Provided the transportation costs relatively decrease over time, these observations suggest a U-shaped relationship between the decrease in transportation costs and spatial agglomeration. That is, dispersion takes place for high and low levels of transportation costs, while agglomeration occurs at intermediate levels. There are several studies describing these phenomena, such as Helpman [4], Krugman and Venables [7], Venables [15], and Puga ([11, 12]).<sup>1</sup> Although the assumptions of these studies vary somewhat, they show a U-shaped relationship between the decrease in transportation costs and spatial agglomeration.

These studies explain the reasons for the agglomeration and dispersion mechanism roughly as follows. When transportation costs are high enough, firms disperse to meet the final demand of peasants in each region; and when transportation costs become intermediate, firms agglomerate to enjoy forward-backward linkages of Marshallian externalities. However, when transportation costs decrease sufficiently, agglomeration is no longer

<sup>1</sup>Also see Mun [10] and Morisugi et al. [9] although their model structures are different, and see Fujita and Thisse [3] and Fujita and Mori [2] for detailed overviews on agglomeration and dispersion of economic activities.

important, since access to other firms and consumers is very easy. That is, a sufficient decrease in interregional transportation costs of manufacturing products nullifies the Marshallian externalities.

There is another line of studies originating in Alonso [1] and extensively developed in the field of urban economics. Consumers use space for housing at a location and commute to the city center in a monocentric city setting. Land market equilibrium yields land rent, land use, and population density as functions of distance from the city center. Henderson [5] extended the single city model to a model of the system of cities. The existence of commuting costs and housing space consumption generates agglomeration diseconomies of congestion, especially in large cities. However, there are also localization economies due to agglomeration of firms in the same industries. In equilibrium, each city specializes in producing an export good and trade goods between cities.

It should be noted that in Henderson's model, interregional (i.e., interurban) transportation costs are assumed to be zero, while intraurban commuting costs are not. To avoid agglomeration diseconomies of congestion, but so as to enjoy localization economies of manufacturing production, each good is produced in one city in equilibrium. Such a simple and extreme result is attributed to the assumption of costless interregional transportation, which may be unrealistic.

On the other hand, in Krugman's model, interregional transportation costs are positive, while intraurban commuting costs are ignored. So, contrary to Henderson's model, each firm producing a differentiated product tends to agglomerate to avoid the costly interregional transportation. This is a major reason for urban agglomeration in Krugman's model, which is the substantive extension. Unfortunately, however, intraurban commuting costs are neglected, which may also be unrealistic.

Since these models are two extreme cases, we unify them in this paper. That is, we assume positive interurban costs and positive intraurban costs. In this sense, this paper is a synthesis of theories by Alonso-Henderson and Krugman.<sup>2</sup>

The objective of this paper is to examine, by using a unified model, possible causes for concentration and dispersion of firms and workers between regions. In Section 2, we start with Krugman's model by adding the element of land consumption of Alonso's model. Incorporation of land enables us to take account of the impacts of price mechanism in the land rent markets on urban concentration and dispersion. Analytical results derived from the model are shown in Section 3, and numerical calcula-

<sup>2</sup> Helpman [4] also introduces housing as a dispersion force. However, it is introduced in a reduced form without considering a full intra-city model. The agricultural sector is also disregarded.

tions and some economic implications are given in Section 4. Section 5 concludes.

## 2. THE MODEL

There are two regions, each containing a CBD (central business district) with negligible space on a featureless plain. Homogeneous workers live around the CBD and commute to it. The utility function of a representative worker living in region  $k$  is expressed as

$$U_k = C_{Mk}^\mu C_{Sk}^\gamma C_{Ak}^{1-\mu-\gamma} \quad (1)$$

and

$$C_{Mk} = \left( \sum_{i=1}^N c_{ik}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \quad \text{for } k = 1, 2,$$

where  $c_{ik}$  is the consumption of manufacturing good  $i$  in region  $k$ ,  $C_{Sk}$  is the consumption of housing space in region  $k$ ,  $C_{Ak}$  is the consumption of agricultural products in region  $k$ . Assuming that agricultural products can be transported without incurring costs, the price of agricultural products is determined by an international market, and is a numéraire. The parameters  $\mu$  and  $\gamma$  are positive with  $\mu + \gamma < 1$ , and the elasticity of substitution is  $\sigma$  with  $\sigma \geq 1$ . There are  $N$  differentiated products. Specifying the CES utility function implies that workers prefer product variety *ceteris paribus*.

Assume Samuelson's iceberg form of interregional transportation costs: a fraction of the good  $\tau \in [0, 1]$  arrives in another region. In other words, the c.i.f. price of the good is  $1/\tau$  times as expensive as the f.o.b. price. If  $p_{ik}$  is the f.o.b. price of good  $i$  in region  $k$  ( $= 1, 2$ ), then the c.i.f. price of good  $i$  in the other region is  $p_{ik}/\tau$ . It should be noted that  $\tau$  is an inverse index of transportation costs.

Suppose there are  $N_k$  firms in region  $k$ . Then, the income constraint for a representative worker in region 1 is given by

$$\sum_{i=1}^N p_{i1} c_{i1} + \sum_{i=N_1+}^N p_{i2} c_{i2} / \tau + r(x) C_{S1} + C_{A1} + T(x) = w_1, \quad (2)$$

where  $x$  is the distance from the CBD,  $r(x)$  is the land (housing) rent at location  $x$ ,  $T(x)$  is the generalized cost of commuting, and  $w_1$  is the wage rate in region 1. The income constraint in region 2 is defined similarly. We assume that the commuting cost is increasing in the commuting distance. We also assume that workers rent housing from absentee landowners, who keep the rental revenue.

Maximizing the utility (1) with respect to  $c_{ik}$  subject to the income constraint (2), we have

$$\frac{c_{i1}}{c_{i2}} = \left( \frac{p_{i2}}{p_{i1}\tau} \right)^\sigma. \quad (3)$$

The total number of population is normalized to 1, and the number of peasants in each region is fixed and given by  $(1 - \mu)/2$ . Denote the number of manufacturing workers in region  $k$  by  $L_k$ , then  $L_1 + L_2 = \mu$  holds. While the peasants are completely immobile, manufacturing workers freely migrate according to the utility difference between two regions in the long run.

The manufacturing production of an individual differentiated good  $i$  involves a fixed cost and a constant marginal cost:

$$l_{ik} = \alpha + \beta c_{ik},$$

where  $l_{ik}$  is the labor input for good  $i$  in region  $k$  and  $c_{ik}$  is the output of good  $i$  in region  $k$ . Each firm maximizes its net profit  $p_{ik}c_{ik} - w_k(\alpha + \beta c_{ik})$  with respect to  $p_{ik}$ , given the constant elasticity of substitution  $\sigma$  in a monopolistic-competition market. The first-order conditions yield

$$\frac{p_2}{p_1} = \frac{w_2}{w_1}. \quad (4)$$

Note that because each firm is symmetric in a differentiated product space, equilibrium prices and quantities are considered to be symmetric too. So we omit subscript  $i$  hereafter.

In addition, assuming free entry and exit of firms, the net profit becomes zero in equilibrium. Manipulating the above equations, we have the ratio of the indirect utility functions<sup>3</sup>:

$$\frac{U_1}{U_2} = \frac{w_1 - T(x_1)}{w_2 - T(x_2)} \left[ \frac{fw_1^{1-\sigma} + (1-f)(w_2/\tau)^{1-\sigma}}{f(w_1/\tau)^{1-\sigma} + (1-f)w_2^{1-\sigma}} \right]^{\mu/(\sigma-1)}, \quad (5)$$

where  $x_k$  is the distance between the CBD and the city border and  $f \equiv L_1/(L_1 + L_2) \in [0, 1]$ . At the city border, the land rent meets the agricultural rent  $r_A$ , which is constant everywhere because of zero trans-

<sup>3</sup>Maximizing the utility (1) with respect to  $c_k$ ,  $C_{Sk}$ , and  $C_{Ak}$  subject to the income constraint (2), we have  $\mu[w_1 - T(x)] = N_1 p_1 c_1 + N_2 p_2 c_2 / \tau$ . From this equation, together with (3) and (4),  $c_k$  and, hence,  $C_{Mk}$  can be expressed by  $w_k$ ,  $T(x)$ , and the parameters. Substituting  $C_{Mk}$ ,  $C_{Sk}$ , and  $C_{Ak}$  into (1) to obtain an indirect utility function, and evaluating it at  $x = x_k$  (since the land rent is the same at each city border), we get the utility ratio of (5).

portation cost of agricultural products. Namely,  $r(x_k) = r_A$  holds. Notice that a long-run equilibrium is attained when (5) is equal to 1.

Now let us introduce the consumption of land for housing, which is lacking in Krugman's [6] model. In a monocentric city setting, each consumer maximizes the utility with respect to the agricultural goods, the manufacturing goods, the housing space, and the location subject to the income constraint. From the first-order conditions, we obtain the well-known location equilibrium condition:

$$r'(x)C_S(x) + T'(x) = 0.$$

It shows that the marginal change in land rent expenditure is offset by the marginal change in commuting costs at each location  $x$ . Here, subscript  $k$  is omitted unless necessary.

Eliminating the variables  $c_i$  and  $C_A$  by the first-order conditions, we have

$$\frac{r(x)C_S(x)}{\gamma} + T(x) = w.$$

From these two equations, we have

$$\log(w - T(x)) = \gamma \log r(x) + \text{constant}.$$

The rent curve is therefore given by

$$r(x) = r_o(1 - T(x)/w)^{1/\gamma}, \quad (6)$$

where  $r_o$  is the land rent at the CBD. We confirm that the land rent is decreasing with distance from the CBD.

The population density at location  $x$  is given by the inverse of the per capita space for housing:

$$\frac{1}{C_S(x)} = \frac{r_o(1 - T(x)/w)^{1/\gamma-1}}{\gamma w}. \quad (7)$$

The population between  $x$  and  $x + dx$  is  $2\pi x dx / C_S(x)$ . Integrating this over the region yields the urban population. Since the urban population is equal to the number of manufacturing workers in each region  $k$ , we obtain

the equilibrium number of manufacturing workers,

$$L_k = \int_0^{x_k} \frac{2\pi x}{C_{S_k}(x)} dx = \frac{2\pi r_A \int_0^{x_k} x(1 - T(x)/w_k)^{1/\gamma-1} dx}{\gamma w_k (1 - T(x_k)/w_k)^{1/\gamma}} \quad \text{for } k = 1, 2. \tag{8}$$

Note that (8) is derived using (6) and (7), in which  $x_k$  substitutes for  $x$  and  $r_A$  substitutes for  $r(x_k)$ .

The total income in each region is given by

$$Y_k = \frac{1 - \mu}{2} + \varphi_k w_k L_k \quad \text{for } k = 1, 2, \tag{9}$$

where

$$\varphi_k \equiv \frac{\int_0^{x_k} x(1 - T(x)/w_k)^{1/\gamma} dx}{\int_0^{x_k} x(1 - T(x)/w_k)^{1/\gamma-1} dx} \quad \text{for } k = 1, 2. \tag{10}$$

$\varphi_k$  is defined by the ratio of the disposable wage to the gross wage, where the disposable wage is the wage net of the commuting costs. This is because the total income net of the commuting costs in region  $k$  is computed as  $\int_0^{x_k} [w_k - T(x)] 2\pi x / C_{S_k}(x) dx = \varphi_k w_k L_k$ . The value  $\varphi_k$  is therefore between zero and one.

Now, following Krugman [6], define  $z_{1k}$  as the ratio of region  $k$  expenditure on region 1 products to that on region 2 products for  $k = 1, 2$ . Using (3) and (4),  $z_{1k}$  can be written as follows:

$$z_{11} = \frac{L_1}{L_2} \left( \frac{w_1 \tau}{w_2} \right)^{-\sigma+1}, \tag{11a}$$

$$z_{12} = \frac{L_1}{L_2} \left( \frac{w_1}{w_2 \tau} \right)^{-\sigma+1}. \tag{11b}$$

The total income of region  $k$  workers is equal to the total consumer spending on region  $k$  manufactures in both regions, i.e.,

$$w_1 L_1 = \mu \left[ \left( \frac{z_{11}}{1 + z_{11}} \right) Y_1 + \left( \frac{z_{12}}{1 + z_{12}} \right) Y_2 \right], \tag{12a}$$

$$w_2 L_2 = \mu \left[ \left( \frac{1}{1 + z_{11}} \right) Y_1 + \left( \frac{1}{1 + z_{12}} \right) Y_2 \right], \tag{12b}$$

where the first term of each right-hand side is the total consumption of manufacturing products in region 1 and the second one is that in region 2.

Setting (5) equal to 1 and plugging  $\varphi_k$  of (10) into (9), we have the system of nine equations of (5)–(9) determining the nine variables  $f$ ,  $x_k$ ,  $w_k$ ,  $Y_k$ , and  $z_{1k}$  for  $k = 1, 2$  in general equilibrium. Obviously, since the system of equations is highly nonlinear, we cannot obtain general solutions analytically in explicit form.

### 3. SOME RESULTS

Our main focus is the impact of the interregional transportation cost  $\tau$  on the city–system structure. Because of the complexity of the equation system, however, analytical results are limited to cases of infinite transportation costs ( $\tau = 0$ ) and zero transportation costs ( $\tau = 1$ ). Since the interregional transportation costs decrease over time historically, the case of low  $\tau$  would correspond to ancient times, and the case of high  $\tau$  would correspond to modern times. In particular, we examine the equilibrium stability of urban concentration and dispersion. In this paper, stable equilibrium means that defection to another city is not profitable for any single firm.

#### 3.1. Infinite Transportation Costs ( $\tau = 0$ )

Let us begin with the case of infinite transportation costs, where autarky prevails in each city.

**PROPOSITION 1.** *When the interregional transportation costs go to infinity, urban concentration is a stable equilibrium if*

$$\mu > \frac{\sigma - 1}{\sigma}. \quad (13)$$

*Proof.* Setting  $f = 1$  in (5), we have

$$\frac{U_1}{U_2} = \left( \frac{w_1 - T(x_1)}{w_2} \right) \tau^{-\mu}. \quad (14)$$

Note that using (9), (12a), and (12b) with  $L_1 = \mu$  and  $L_2 \rightarrow 0$ ,  $w_k$ 's are expressed as

$$w_1 = \frac{1 - \mu}{1 - \varphi_1 \mu} \quad \text{and}$$

$$w_2 = \left( \frac{1 - \mu}{1 - \varphi_1 \mu} \right)^{(\sigma-1)/\sigma} \times \left[ \tau^{\sigma-1} \left( \frac{1 - \mu}{2} + \frac{1 - \mu}{1 - \varphi_1 \mu} \varphi_1 \mu \right) + \tau^{-\sigma+1} \left( \frac{1 - \mu}{2} \right) \right]^{1/\sigma}.$$

It is easily shown that if  $\tau$  approaches 0, (14) becomes  $+\infty$  under the condition of (13). This implies that manufacturing concentration is a stable equilibrium. ■

Equation (13) shows that in the case of prohibitively high transportation costs of manufacturing goods (i.e.,  $\tau \rightarrow 0$ ), firms concentrate when (i) the elasticity of substitution  $\sigma$  is low, and (ii) the ratio of manufacturing employment (and that of manufacturing expenditure)  $\mu$  is high. The former implies that when the substitutability between manufacturing goods is low under very high transportation costs, the consumer's utility in a small city must be very low, since importing less substitutable goods is prohibitively costly. That is, living in a desert is not attractive at all for consumers, whereas living in a large city is attractive because of a variety of differentiated products. Implications of the latter ( $\mu$ ) are straightforward. Agglomeration takes place when the manufacturing is important relative to the agriculture and housing.

When the inequality in (13) is reversed, we conjecture that even dispersion is a stable equilibrium. It is true according to our various numerical computations. However, we can prove it analytically only in the case of  $\gamma = 1/2$  and  $T(x) = tx$ , where  $t$  is a positive constant, while Proposition 1 holds for any value of  $\gamma \in (0, 1)$  and for any  $T(x)$ .

**PROPOSITION 2.** *When the interregional transportation costs go to infinity and  $\gamma = 1/2$ , even dispersion is a stable equilibrium if*

$$\mu \leq \frac{\sigma - 1}{\sigma}. \quad (15)$$

*Proof.* See the Appendix. ■

In this subsection, the interregional transportation costs are prohibitively high. This corresponds to ancient times, where there is no means of transportation between regions. Each urban economy is autarkic, and the city size distribution is determined by the distribution of peasants. So, even dispersion as in Proposition 2 must have taken place in ancient times, rather than urban concentration, as in Proposition 1. That is, initially (i) the elasticity of substitution  $\sigma$  was high, and (ii) the ratio of manufacturing employment  $\mu$  was low relative to agricultural employment. Thus, we assume  $\mu \leq (\sigma - 1)/\sigma$  as in Proposition 2 hereafter.

### 3.2. Zero Transportation Costs ( $\tau = 1$ )

Next, consider the other polar case of zero interregional transportation costs, where perfect economic integration takes place between cities. This corresponds to modern times or the far future, where interregional transportation is very easy. In this case, we obtain the following strong result.

PROPOSITION 3. *When the interregional transportation costs become zero, even dispersion is the unique stable equilibrium for any value of the parameters.*

*Proof.* Setting  $\tau = 1$  in (5), we have  $U_1/U_2 = (w_1 - T(x_1))/(w_2 - T(x_2))$ . From (11a) and (11b), we get  $z_{11} = z_{12}$  for all  $f \in [0, 1]$ . From (11a), (11b), (12a), and (12b), we obtain  $w_1 = w_2$  for all  $f \in [0, 1]$ . Given  $w_1 = w_2$ ,  $\partial L_k / \partial x_k > 0$  is obvious from (8). Thus, we can derive  $d(U_1/U_2)/df < 0$  for all  $f \in [0, 1]$ . That means  $(L_1, L_2) = (\mu/2, \mu/2)$  is the unique and globally stable equilibrium. ■

Proposition 3 is a main result of the paper. It implies that dispersion is the ultimate state of the city system *for any initial condition and for any parameter values*. When interregional transportation costs become negligible because of technological progress, firms and workers will be dispersed, and urban agglomeration will cease in the far future. In this case, there is no reason to be concentrated. Instead, a dispersed location enables consumers to enjoy greater consumption of land and shorter time of commuting.

It should be noted that the same would be true for information service industries using internets, where interregional transmission of information is comparable to interregional transportation of commodities. Technical progress in telecommunications induces dispersion of urban activities, like progress in transportation.

#### 4. ILLUSTRATION

So far, we have confined our analysis to the polar cases because of the nonlinear system of equations. To understand the model structure further, we have to rely on numerical calculation using specific values of parameters.

Specifying  $T(x) = x$ , consider the case of  $\sigma = 4$ ,  $\mu = 0.3$ ,  $\gamma = 0.5$ , and  $r_A = 10$ . We know from Proposition 2 that when  $\tau = 0$ , even dispersion is an equilibrium in this set of parameters, since the elasticity of substitution  $\sigma$  is high and the manufacturing share  $\mu$  is low enough.

Numerical calculation is conducted in the following manner. Given the above parameter values, we first fix the value of  $f$ , and set initial values of  $x_k$ 's. Then,  $w_k$ 's are determined by (8),  $\varphi$ 's by (10),  $Y_k$  by (9), and  $z_{1k}$ 's by (11a) and (11b). By putting these values into (12a) and (12b), we evaluate the differences between the right-hand sides and left-hand sides. If they are large, we change the values of  $x_k$ 's and repeat the same calculation procedure until the differences become small enough.

Now, setting (5) equal to 1, we obtain a collection of equilibria. Eliminating unstable ones, we have the stable equilibrium distribution of

manufacturing workers according to the transportation cost parameter  $\tau$  as follows:

$$\begin{aligned}
 (L_1, L_2) &= (\mu/2, \mu/2) && \text{for } 0 \leq \tau < 0.52 && \text{[case (a)]} \\
 &= (\mu/2, \mu/2), (\mu_1, \mu - \mu_1), (\mu - \mu_1, \mu_1) && && \\
 &&& \text{for } 0.52 < \tau < 0.53 && \text{[case (b)]} \\
 &= (0, \mu), (\mu, 0) && \text{for } 0.53 < \tau < 0.83 && \text{[case (c)]} \\
 &= (\mu_2, \mu - \mu_2), (\mu - \mu_2, \mu_2) && && \\
 &&& \text{for } 0.83 < \tau < 0.89 && \text{[case (d)]} \\
 &= (\mu/2, \mu/2) && \text{for } 0.89 < \tau \leq 1, && \text{[case (e)]}
 \end{aligned}$$

where  $\mu_1$  and  $\mu_2$  vary in  $(0, \mu/2)$  as  $\tau$  changes. Cases (a)–(e) correspond to those in Fig. 1. To visualize the transition of stable equilibria due to the change in  $\tau$ , we depict the utility level in each region with respect to region 1's employment distribution  $f$  in Fig. 1.

In Fig. 1, the utility curves are U-shaped when the transportation costs are large (i.e.,  $\tau$  is small). The left side of  $U_1$  shows the phase of negative externalities in that larger city size decreases per capita housing space and increases commuting costs. On the other hand, the right side of  $U_1$  shows the phase of positive externalities in that larger city size increases product variety. When the costs of transporting goods become small, such positive externalities tend to vanish, since agglomeration is no longer necessary under small interregional transportation costs. So the utility curve becomes downward sloped at the right side. These changes in the utility curves alter the equilibrium distribution of firms and workers.

The existence of multiple equilibria means indeterminacy of the state. However, we can refine such multiple equilibria if we assume a monotonic change in the transportation parameter  $\tau$ . We start from the case of very high transportation costs ( $\tau = 0$ ) and decrease the costs continuously up to zero transportation costs ( $\tau = 1$ ).

In doing so, we can exclude the asymmetric distribution in case (b), which requires a discontinuous jump in the equilibrium established under case (a). The equilibrium distribution of manufacturing firms/workers is then slightly simplified as

$$\begin{aligned}
 (L_1, L_2) &= (\mu/2, \mu/2) && \text{for } 0 \leq \tau < 0.53 \\
 &= (0, \mu), (\mu, 0) && \text{for } 0.53 < \tau < 0.83 \\
 &= (\mu_2, \mu - \mu_2), (\mu - \mu_2, \mu_2) && \text{for } 0.83 < \tau < 0.89 \\
 &= (\mu/2, \mu/2) && \text{for } 0.89 < \tau \leq 1.
 \end{aligned}$$

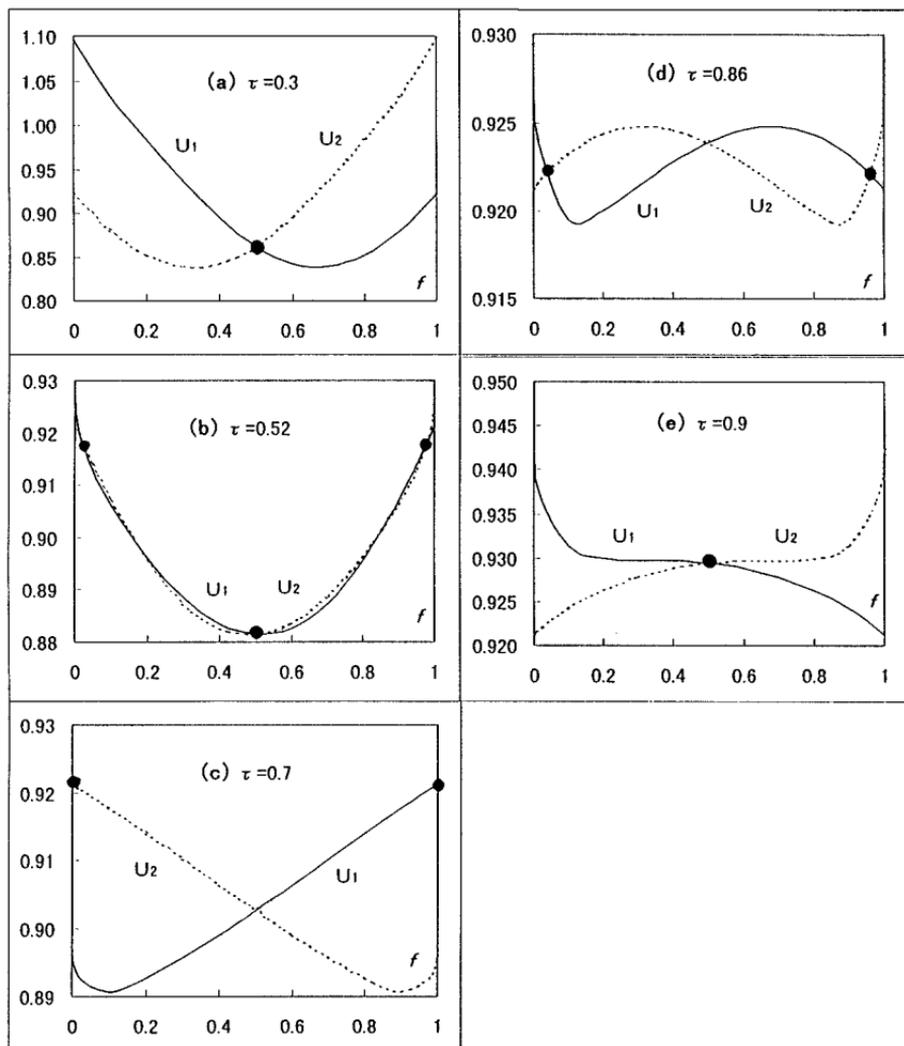


FIG. 1. Transition of equilibrium due to the change in the transportation costs.

In Fig. 2, the manufacturing distribution is drawn according to the continuous change in the transportation cost parameter  $\tau$ . It shows that *dispersion of firms and workers takes place for the small or large transportation costs, whereas concentration occurs for the intermediate transportation costs.*<sup>4</sup> We may interpret this finding in the following manner.

<sup>4</sup>If  $\mu > (\sigma - 1)/\sigma$ , as in the case of Proposition 1, then we start from the manufacturing concentration. The concentration would continue during the large transportation costs, while dispersion would take place when the transportation costs get small. That is, only cases (c), (d), and (e) in Fig. 1 are realized for  $\mu > (\sigma - 1)/\sigma$ .

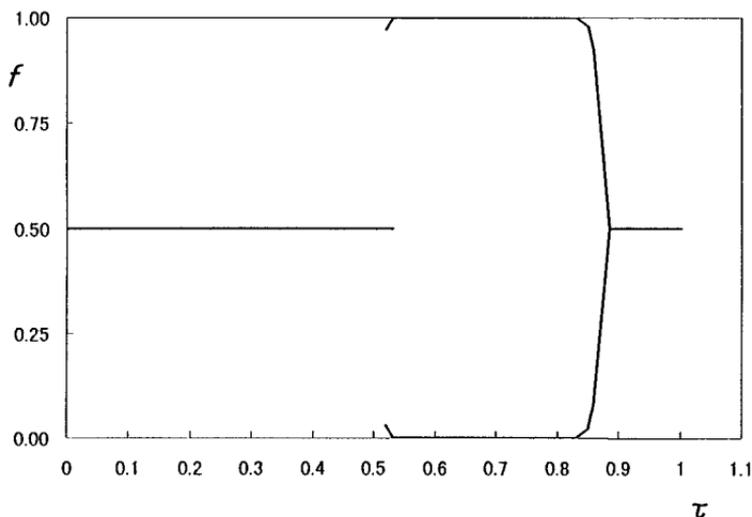


FIG. 2. Equilibrium manufacturing distribution due to the change in the transportation costs.

*Cases (a) and (b).* When the cost of transporting goods is sufficiently high, interregional trade seldom takes place, and each region is nearly self-sufficient. In such a case, the utility level in each region is determined mainly by the amount of housing space and the variety of manufacturing goods within the region. In a small city, since a higher proportion of the manufacturing goods should be imported with high transportation costs, the price index must be high. This would raise the wage rate. So workers will consume more agricultural products and more housing space in a small city. Because the prices and the wage rate rise proportionally according to Eq. (4), workers must be better off.<sup>5</sup> The reverse is true in a region with many firms and workers (i.e., a large city). Since there are a variety of manufacturing goods, their prices are lower and the wage rate is lower, and hence workers consume fewer agricultural products and less space, leading to a lower utility level.

It should be noted that under these circumstances the large size of the export industry does not enhance the regional welfare level. Instead, workers in the small city are well off (and peasants in the large city are

<sup>5</sup>The reader should pay attention to the counterintuitive Lemma 2, showing  $dx_1/df < 0$ , the smaller the manufacturing agglomeration is, the farther from the CBD the city border is. When the transportation costs are very large, the wage and prices are high in the city with a small number of workers. However, since there is less variety of manufacturing products, their expenditure goes to the consumption of vast housing space. Surprisingly, this effect is so strong that aggregate urban space for housing is greater in the city with a smaller number of workers.

well off). The discussion is valid in the short run, where the workers and firms cannot migrate between regions. However, since they migrate costlessly from the large city to the small city in the long run, both regions become of equal size in the long-run equilibrium.

*Case (c).* When the transportation cost decreases such that the parameter  $\tau$  exceeds a critical value (0.53), sudden agglomeration takes place, i.e., manufacturing firms and workers migrate to one region. A decrease in the transportation costs encourages firms in the large city to export their manufacturing goods, and tends to diminish the price differentials and hence the wage differentials. In the large city, variety in manufacturing goods (agglomeration force) becomes more important than scarcity in residential land (dispersion force), leading to an increase in the utility level. Thus agglomeration becomes stable. Note that while the change in  $\tau$  is continuous, catastrophic agglomeration takes place at the critical value.

*Case (e).* When the transportation cost becomes sufficiently low, however, the benefits of urban agglomeration vanish, as mentioned before. The constraint of residential land outweighs the urban agglomeration economies, leading to re-dispersion of firms and workers. When the transportation cost parameter  $\tau$  approaches one, which is equivalent to zero transportation cost, the location of production and consumption of manufacturing goods does not matter any longer. The only concern in the location decision is the space for housing. Thus, as shown in Proposition 3, firms and workers will be re-dispersed in the far future.

Until now, we considered the change in the interregional transportation cost  $\tau$  while the intraregional transportation cost  $t$  kept constant. It may be natural to think that technological progress reduces the intraregional transportation cost as well, which might hinder the future trend of re-dispersion. However, the reduction in the intraregional transportation cost is limited to a certain extent, since it is commuting (Trefil [14]). Indeed, technological progress would decrease the interregional transportation costs of commodities, but it would decrease the intraregional transportation costs of commuters little. This is due to the physical constraints of rush-hour congestion of roads and trains, which cannot be substantially reduced without the advent of an ultrarapid mass transit generating little congestion. In other words, compared to pecuniary costs of commodity flows, time costs of commuting (opportunity costs of time) are difficult to overcome. Hence, Proposition 3 would still be valid and important.

Finally, we would like to mention welfare considerations. Since the profits of the firms become zero, we pay attention to the utility level of workers. As the transportation cost decreases, the equilibrium utility level increases continuously. However, there is an exception at the critical value ( $\tau = 0.53$ ) when the sudden agglomeration occurs. In this instance, the equilibrium utility level rises catastrophically. In practice, we may say that

since abrupt urbanization due to rural–urban migration enhances the level of utility, it should not be restricted.

In Fig. 1, when the interregional transportation costs are large, the utility curves are U-shaped. It implies that even dispersion is worse than agglomeration from a social welfare point of view. In fact, the utility level of the agglomerated equilibrium is higher than that of the dispersed equilibrium in the case of multiple equilibria [case (b)]. Therefore, decentralization policies, which are often conducted in regional planning, are not justified in the presence of large transportation costs. In this case, product variety due to urban agglomeration is of importance.

However, when the interregional transportation costs become small, the utility curves become downward sloped [case (e)]. In this case, dispersion is better than agglomeration. Since the even dispersion equilibrium is unique and stable, it is attained without any government intervention. Consequently, we may say that given the specifications and the parameter values, *agglomeration policies are desirable in the process of urbanization, but no policies are necessary in the process of decentralization.*

## 5. CONCLUSION

In this paper, we incorporated the standard intracity resource allocation framework into Krugman's [6] intercity allocation method with differentiated products. We presented a general equilibrium model of agglomeration and dispersion of firms and workers in a two-city system framework under the presence of positive externality (product variety) and negative externality (congestion). We obtained the conditions of agglomeration and dispersion in the case of very high interregional transportation costs in Propositions 1 and 2. We then showed in Proposition 3 that dispersion always occurs when the interregional transportation costs become sufficiently low.

We also conducted a numerical analysis by using particular parameter values, and illustrated a transition from dispersion to agglomeration, and then re-dispersion when the transportation cost decreases monotonically. We found that the welfare level in the dispersed state is usually lower than that in the agglomerated state, and that agglomeration policies should be made in the developing stages, whereas no policies are necessary in the developed states.

## APPENDIX

*Proof of Proposition 2.* We begin with three lemmas.

LEMMA 1. *When  $\tau = 0$ ,  $f = 1/2 + \varepsilon$ , and  $\gamma = 1/2$ ,*

$$\frac{dw_1}{dT_1} > \frac{w_1}{T_1}$$

*holds for arbitrarily small  $\varepsilon > 0$ , where  $T_k \equiv tx_k$ .*

*Proof.* Since the integrals in Eqs. (8) and (10) can be solved explicitly, manipulating them with  $\tau = 0$ ,  $\gamma = 1/2$  and Eqs. (12a) and (9) to eliminate  $f$ , we have

$$w_1 = \frac{2(1 - \mu) + 3AT_1^2}{2(1 - 2\mu)f}, \quad (16)$$

$$H \equiv \left[ Aw_1 \left( \frac{w_1}{2} + T_1 \right) + \frac{\mu}{2 - \mu} \left( 1 - \mu + \frac{3}{2} AT_1^2 \right) \right] (w_1 - T_1)^2 - \frac{Aw_1^4}{2} = 0, \quad (17)$$

where  $A \equiv 2\pi r_A / [(1 + \gamma)t^2]$ .

Differentiating  $H$  yields

$$\left. \frac{\partial H}{\partial w_1} \right|_{H=0} = \frac{AT_1^3(2w_1 - T_1)}{w_1 - T_1} > 0,$$

$$\left. \frac{\partial H}{\partial T_1} \right|_{H=0} = \frac{A}{w_1 - T_1} \left[ \left( w_1 + \frac{3\mu}{2 - \mu} T_1 \right) (w_1 - T_1)^3 - w_1^4 \right].$$

By use of the implicit function theorem,

$$\frac{dw_1}{dT_1} = - \frac{\{w_1 + [3\mu/(2 - \mu)]T_1\}(w_1 - T_1)^3 - w_1^4}{T_1^3(2w_1 - T_1)}.$$

Therefore, proving Lemma 1 is equivalent to proving the following:

$$T_1^3(2w_1 - T_1)w_1 + \left[ \left( w_1 + \frac{3\mu}{2 - \mu} T_1 \right) (w_1 - T_1)^3 - w_1^4 \right] T_1 < 0,$$

since  $\partial H / \partial w_1 > 0$ . Dividing the LHS by  $T_1 w_1^4$  and defining  $v \equiv T_1 / w_1 \in (0, 1)$ , it becomes

$$v^2(2 - v) + \left( 1 + \frac{3\mu}{2 - \mu} v \right) (1 - v)^3 - 1$$

$$= \frac{(1 - v)v}{2 - \mu} [6(\mu - 1) + 4(1 - 2\mu)v + 3\mu v^2].$$

The terms in square brackets are convex with respect to  $v$ , and its sign is negative at  $v = 0, 1$ . This means that the sign is negative for all  $v \in (0, 1)$ , and hence Lemma 1 is proved. ■

LEMMA 2. When  $\tau = 0$ ,  $f = 1/2 + \varepsilon$ , and  $\gamma = 1/2$ ,

$$\frac{dx_1}{df} < 0$$

holds for arbitrarily small  $\varepsilon > 0$ .

*Proof.* Eliminating  $w_1$  from Eqs. (16) and (17), we get

$$(2 - \mu) \left[ T_1(4\mu f + 3AT_1) + \sqrt{aT_1^3(8\mu f + 9AT_1)} \right] - 2[2\mu(1 - \mu) + 3\mu AT_1^2] = 0.$$

Differentiating it with respect to  $f$  and  $T_1$ , respectively, and applying the implicit function theorem, we can show directly that  $dT_1/df = tx_1/df < 0$ .

■

LEMMA 3. When  $\tau = 0$ ,  $f = 1/2 + \varepsilon$ , and  $\gamma = 1/2$ ,

$$\frac{w_1 - tx_1}{w_2 - tx_2} < \frac{w_1}{w_2}$$

holds for arbitrarily small  $\varepsilon > 0$ .

*Proof.* Proving Lemma 3 is equivalent to proving

$$J(f) \equiv w_2T_1 - w_1T_2 > 0$$

for  $f = 1/2 + \varepsilon$ . Differentiating  $J(f)$  yields

$$\begin{aligned} \frac{dJ}{df} \Big|_{f=1/2+\varepsilon} &= \left( \frac{dw_2}{dT_1} T_1 + w_2 - \frac{dw_1}{dT_1} T_2 - w_1 \frac{dT_2}{dT_1} \right) \frac{dT_1}{df} \\ &= \frac{1}{T_1} \left( \frac{dw_1}{dT_1} - \frac{w_1}{T_1} \right) \left( \frac{dT_2}{dT_1} - 1 \right) \frac{dT_1}{df}, \end{aligned}$$

since  $T_2 \rightarrow T_1$ ,  $w_2 \rightarrow w_1$ , and  $dw_2/dT_2 \rightarrow dw_1/dT_1$  for  $f \rightarrow 1/2$ . Now, using Lemmas 1 and 2 with  $dT_2/dT_1 < 0$ , we obtain  $dJ(1/2)/df < 0$ . Since  $J(1/2) = 0$ , Lemma 3 is thus shown. ■

We are now ready to prove Proposition 2. When  $\tau$  approaches 0, Eq. (5) is reduced to

$$\begin{aligned} \frac{U_1}{U_2} \Big|_{f=1/2+\varepsilon} &= \frac{w_1 - tx_1}{w_2 - tx_2} \left( \frac{w_1}{w_2} \right)^{-\mu} \left( \frac{f}{1-f} \right)^{\mu/(\sigma-1)} \\ &< \left( \frac{w_1}{w_2} \right)^{1-\mu} \left( \frac{f}{1-f} \right)^{\mu/(\sigma-1)} \\ &\leq \left[ \frac{w_1 f}{w_2(1-f)} \right]^{1-\mu} \\ &< 1. \end{aligned}$$

The first inequality is due to Lemma 3, and the second one is due to the condition in Proposition 2. The third inequality is due to  $d(w_1 f)/dT_1 > 0$  from (16) and to  $dT_1/df < 0$  from Lemma 2. Therefore, migration from region 1 to region 2 would decrease the migrant's utility level, implying the stability of the even dispersion.

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